Return to Session Directory

DYNAMIC POSITIONING CONFERENCE
October 12-13, 2010

**Design Session**

# Output Feedback Passivity Based Controllers for Dynamic Positioning of Ships

**Shah Muhammad**
*Delft University of Technology, the Netherlands*

**Arnau Dòria-Cerezo**
*Universitat Politècnica de Catalunya, Barcelona, Spain*

## Abstract

In this paper, a family of passivity based controllers for dynamic positioning of ships is presented. We exploit the idea of shaping the energy function of the closed loop system to obtain different formulations of the passivity based control law using the IDA-PBC methodology. A salient feature of this study is that the proposed control laws are output feedback controllers and the relative velocity measurement is not required. First, we design and analyze two static controllers which can be seen as a nonlinear version of the conventional PD controllers. In presence of unknown disturbances, these controllers do not provide the desired regulation properties. To remove this discrepancy we propose, also in the context of the IDA-PBC technique, a dynamic extension of the system and obtain two new controllers which have the desired regulation properties. These new control laws can be seen as a nonlinear version of the conventional PID controllers. Simulations are included to validate the theoretical results.


## 1. Introduction

A dynamic positioning (DP) system is a computer controlled system which automatically maintains a vessel's position and heading by using propellers and thrusters. The computer program contains a mathematical model of the vessel which includes information pertaining to the wind and current drag of the vessel and the location of the thrusters. This knowledge, combined with the sensor information, helps the computer to calculate the required steering angle and thruster output for each thruster. The first DP system was introduced in 1960 and since then they have emerged as a popular alternative for the conventional mooring and anchoring techniques for the dynamical positioning of the ships. Over the past decades, with the revolutionary developments in microprocessor technologies and availability of fast computing machines, DP systems have become more economical and reliable.

Generally, in DP problems, only position and heading measurements are available. This leads to the use of observers to estimate the state (mainly the velocities and the bias term) which are required to feedback into the control law. This problem is studied in many papers. Some examples include the Luengberger observer used in [14], a nonlinear observer designed in [3], or a passivity-based scheme considered in [4] and [9].

Furthermore, the measured position and heading signals are noisy and, also, with two different frequency components. The total ship motion can be seen as a superposition of a low frequency component (due to the wind, sea currents and thruster forces and moments) and an oscillatory term (the so-called wave induced wave frequency motion), which represents the effect of the waves [2].

However, DP only considers the slow variations and, consequently, the motion due to the waves should be removed before it enters in the controller algorithm. Kalman filtering techniques were proposed in [7] and [6], or see [5] for a recent overview. As pointed out in [4], Kalman filters require the use of a linear model, and the nonlinear motion should be linearized at various operation points. To overcome this drawback, in [5] and [9], a wave-frequency observer is added to compensate the wave disturbances.

Due to the important role of the estimation and filtering process, the motion control system in the DP problem can be grouped in two basic subsystems: the observer system (or wave filter), and the controller, see Figure 1.

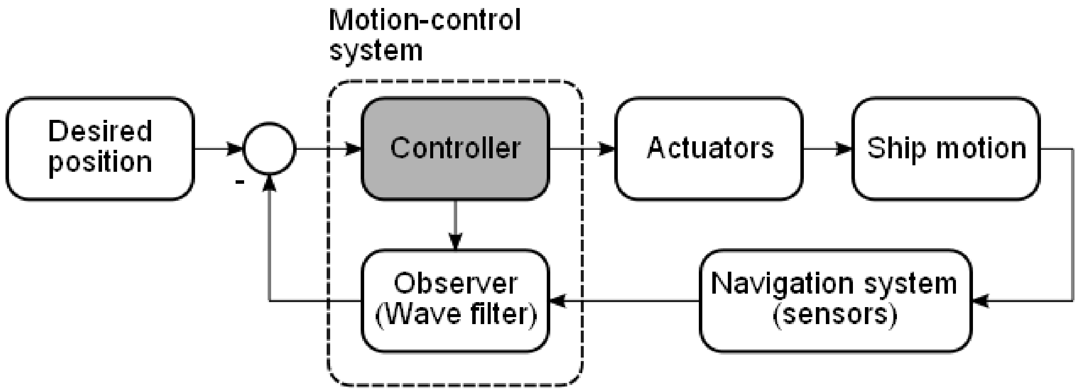


*Figure 1. Basic scheme of components of a ship motion-control system.*

Various controllers have been proposed to stabilize the ship to the desired position. PI controllers are often used [2], however more advanced techniques are applied to this problem resulting in interesting control algorithms. Backstepping design, which also includes the observer stage, are presented in [3] and [13]. In [9] the whole stability of a PD-type controller with a passive observer is proved using a separation principle argument. Recently, sampled-data control theory has also been applied to the DP problem for designing the control law [8].

This paper is focused on the design of the control law assuming that the filtering and observation process are previously done. The main contribution is a family of passivity-based controllers which use the energy shaping of the closed loop system to ensure (local/global) asymptotic stability. Passivity-based techniques have been used in many applications. A nice feature of the passivity-based control design is the physical meaning of the resulting control laws and the concepts such as storage energy or dissipation which play a fundamental role in the stability analysis and performance. Stability properties, based on the Lyapunov theory, can be easily studied for the obtained closed loop systems. In the last decade, the Interconnection and Damping Assignment-Passivity-Based Control (IDA-PBC) methodology has emerged as an easy and a (quasi) step-by-step methodology to obtain passivity-based controllers, see for instance [12].

Two different energy functions are proposed in this paper. We start by illustrating the methodology using a quadratic and a trigonometric Hamiltonian function and recover a simple static controller which guarantees asymptotic stability. The energy shaping, based on a trigonometric function, improves the heading control. These controllers do not produce the desired regulation properties in presence of unknown disturbances. Consequently, in order to achieve the desired performance, a dynamic extension is proposed, and it results in a control law that can be interpreted as a nonlinear version of the conventional PID controller. A salient feature of the proposed controllers is that they do not require the relative velocity measures and, thanks to a dynamic extension, they also ensure a good regulation behavior even in presence of disturbances or unknown (or non-estimated) terms.

The presentation of the contents of this paper is as follows: Section 1 is reserved for the introduction. It explains some of the basic details of the problem under consideration and recalls some existing works on this subject. In Section 2 the port-Hamiltonian framework is introduced and an overview of the IDA-PBC methodology for the design of passivity based controllers is given. Section 3 contains the details of the ship model which we study in this paper and its port-Hamiltonian form is derived. In Section 4, a static controller based on the IDA-PBC methodology is obtained. Two variants, a quadratic and a trigonometric, of this controller are then analyzed. Then, a dynamic extension is proposed to revamp the static controller for an improved performance. Section 6 contains the simulation results to give a qualitative measure of the performance of the proposed controllers and, finally, Section 7 consists of the concluding remarks.

## 2. Hamiltonian Based Control

### 2.1 Port-Hamiltonian modeling

A large class of physical systems of interest in control applications can be modeled in the general form of port- Hamiltonian systems (PHS) [15]. PHS generalize the Hamiltonian formalism of classical mechanics to physical systems connected in a power-preserving way and encodes the detailed energy transfer and storage in the system, and is thus suitable for the control schemes based on the IDA-PBC.

A PHS can be written, in an implicit form, as

$$\dot{\mathbf{x}} = (\mathfrak{I} - \mathcal{R})\,\partial \mathrm{H}(\mathbf{x}) + \mathrm{g}(\mathbf{x})\,\mathbf{u}, \qquad (1)$$

where $\boldsymbol{x} \in \mathbb{R}^n$ is the state (or Hamiltonian variables) vector, $\mathfrak{I}(\boldsymbol{x}) \in \mathbb{R}^{n\times n}$ is the interconnection (skew-symmetric, $\mathfrak{I} = -\mathfrak{I}^{\mathrm{T}}$) matrix , $\mathcal{R}(\mathbf{x}) \in \mathbb{R}^{n\times n}$ is the dissipation matrix (which is symmetric positive semi-definite, $\mathcal{R} = \mathcal{R}^{\mathrm{T}} \geq 0$), $\mathrm{g}(\mathbf{x}) \in \mathbb{R}^{n\times m}$ is the external connection matrix, $\mathbf{u} \in \mathbb{R}^m$ is the control input vector, and $\mathrm{H}(\mathbf{x})$ is the Hamiltonian (or energy) function.

The so-called passive output, $\mathbf{y} \in \mathbb{R}^m$, is given by

$$\mathbf{y} = \mathrm{g}^{\mathrm{T}}(\mathbf{x})\,\partial \mathrm{H}(\mathbf{x}), \qquad (2)$$

and the product $\mathbf{u}^{\mathrm{T}}\mathbf{y}$ has, usually, unity of power.

### 2.2 The IDA-PBC technique

The Interconnection and Damping Assignment-Passivity-Based Control (IDA-PBC), [12], is a technique for designing controllers based on the port-Hamiltonian framework. It uses the passive stability properties to ensure the convergence of the system to the desired fixed point. The main idea behind the IDA-PBC is to define a new closed loop (or target) system with a Hamiltonian structure. The design problem summarizes into finding a control law such that the system behaves as

$$\dot{\mathbf{x}} = (\mathfrak{I}_{\mathrm{d}} - \mathcal{R}_{\mathrm{d}})\,\partial \mathrm{H}_{\mathrm{d}}, \qquad (3)$$

where $\mathfrak{I}_{\mathrm{d}}(\mathbf{x}) = -\mathfrak{I}_{\mathrm{d}}^{\mathrm{T}}, \mathcal{R}_{\mathrm{d}}(\mathbf{x}) = \mathcal{R}_{\mathrm{d}}^{\mathrm{T}} \geq 0$ and $\mathrm{H}_{\mathrm{d}}(\mathbf{x})$ has a minimum at the desired regulation point $, \mathbf{x}^{\mathrm{d}} = \arg\min\,(\mathrm{H}_{\mathrm{d}}(\mathbf{x})$ . The stability of this system can be easily proved by using $\mathrm{H}_{\mathrm{d}}$ as a Lyapunov function ($\dot{\mathrm{H}}_{\mathrm{d}}(\mathbf{x}) = -(\partial \mathrm{H}_{\mathrm{d}})^{\mathrm{T}}\mathcal{R}_{\mathrm{d}}\partial \mathrm{H}_{\mathrm{d}} \leq 0$, see for instance, [11] and [12] for a detailed discussion).

The design procedure reduces to finding matrices $\mathfrak{I}_{\mathrm{d}}(\mathbf{x})$ and $\mathcal{R}_{\mathrm{d}}(\mathbf{x})$ and a desired closed loop energy function $\mathrm{H}_{\mathrm{d}}(\mathbf{x})$, which solve the so-called matching equation

$$(\mathfrak{I} - \mathcal{R})\,\partial \mathrm{H} + \mathbf{gu} = (\mathfrak{I}_{\mathrm{d}} - \mathcal{R}_{\mathrm{d}})\partial \mathrm{H}_{\mathrm{d}} \qquad (4)$$

Then, the control law becomes

$$\mathbf{u} = (g^T g)^{-1} g^T \left( (\mathfrak{J}_d - \mathcal{R}_d) \partial H_d - (\mathfrak{J} - \mathcal{R}) \partial H \right) \tag{5}$$

A drawback of the IDA-PBC controllers is that they are, in general, not able to reject disturbances. To remove this discrepancy of the control design usually a dynamic extension of the system is done to obtain an integral action on the output error. Extension of the closed loop dynamics in the IDA-PBC framework can be done, in a natural way, only for passive outputs, [11]. A completely different problem addresses for non-passive outputs (or higher relative degree one outputs). In this case, a Hamiltonian based controller with an integral action can be obtained via a change of variables, [1]. For more mathematical details readers are referred to [10].

## 3. The Ship Model

A useful model describing the dynamics of a surface ship sailing in a horizontal plane having degrees of freedom, is given in [2], and it can be written as the following nonlinear system

$$\begin{bmatrix} \dot{\boldsymbol{\eta}} \\ \dot{\mathbf{v}} \end{bmatrix} = \begin{bmatrix} O_3 & J(\psi) \\ O_3 & -M^{-1}D \end{bmatrix} \begin{bmatrix} \boldsymbol{\eta} \\ \mathbf{v} \end{bmatrix} + \begin{bmatrix} O_3 \\ M^{-1} \end{bmatrix} \tau + \begin{bmatrix} O_3 \\ M^{-1}J^T(\psi) \end{bmatrix} \mathbf{b}, \tag{6}$$

where $\boldsymbol{\eta} = [x \quad y \quad \psi]^T$ is the position coordinate vector in the Earth-fixed reference frame, $\mathbf{v} = [u \quad v \quad r]^T = [\dot{x}_r \quad \dot{y}_r \quad \dot{\psi}_r]^T$ is the relative vessel-frame velocity coordinate vector, $\boldsymbol{\tau} = [\tau_u \quad \tau_v \quad \tau_r]^T$ is the vector describing the forces and the torque in vessel-fixed reference frame provided by the propulsion system of the ship acting in the *surge*, *sway* and *yaw* directions, respectively, and

$$J(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

is the coordinate transformation matrix which relates the Earth-fixed frame to the relative-frame of reference. A description of the two frames of reference is given in Figure 2. The D and M matrices are given by

$$D = \begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & d_{23} \\ 0 & d_{32} & d_{33} \end{bmatrix}, M = \begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & m_{23} \\ 0 & m_{32} & m_{33} \end{bmatrix},$$

which are positive definite, $D = D^T > 0$ and $M = M^T > 0$, and $O_3$ is a $3 \times 3$ zero matrix. The environmental disturbances due to the sea currents, waves, and wind are represented by $\mathbf{b} = [b_1 \quad b_2 \quad b_3]^T$ in the Earth-fixed reference frame. This bias term is constant in the Earth-fixed reference frame, under assumption of constant or slowly varying currents.

Along this paper, it is assumed that these nature effects (sometimes called as bias forces and moments), which can also be modeled as a first-order Markov process, [2], are either known or an estimate of the bias vector is available. Adding a note on the output of the system, we consider that the measurement system gives us noise free position and orientation measurements and that the wave frequency (WF) components from the measured output are filtered or estimated. Hence, in this paper, we skip the dynamics of the bias and the WF components.

The main goal in the dynamic positioning problem is to stabilize the ship in a given $\eta$ −coordinate. Without loss of generality, our objective is to design an appropriate control law $\boldsymbol{\tau}$

which stabilizes the system to the origin $(x, y, \psi) = (0,0,0)$. Additionally, as the measurement of the relative velocity vector is not available, the control law should be independent of $\mathbf{v}$, and must be able to reject unknown disturbances or uncertainties.

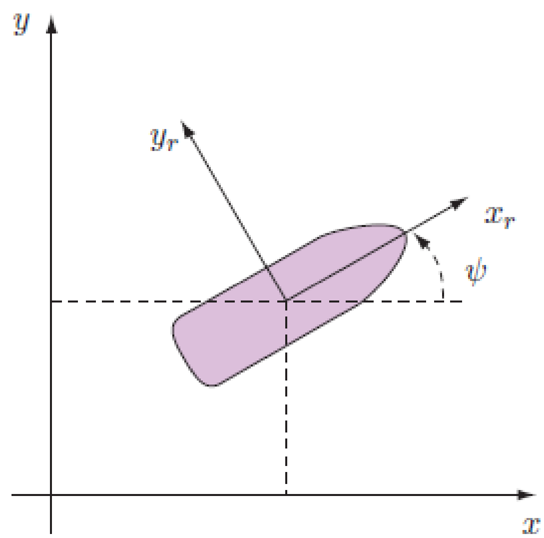


*Figure 2. Description of the earth-fixed and the vessel-fixed frames of reference.*

We can write the system described in (6) in a PHS form (1) by using as a state $\mathbf{x}^T = [q^T \quad p^T] \in \mathbb{R}^6$, where $\mathbf{q} = [q_1 \quad q_2 \quad q_3]^T \in \mathbb{R}^3$ represents the Earth-fixed position and heading, and the momentum $\mathbf{p} = [p_1 \quad p_2 \quad p_3]^T \in \mathbb{R}^3$, is defined as $\mathbf{p} = M\mathbf{v}$. Substituting $\boldsymbol{\eta} = \mathbf{q}$ and $\mathbf{v} = M^{-1}\mathbf{p}$ in (6), we get the following system

$$\dot{\mathbf{x}} = (\mathfrak{J}(q_3) - \mathcal{R})\,\partial H + g_\tau \boldsymbol{\tau} + g_b(q_3)\mathbf{b} \tag{7}$$

with the following interconnection and damping matrices

$$\mathfrak{J}(q_3) = \begin{bmatrix} O_3 & J(q_3) \\ J^T(q_3) & O_3 \end{bmatrix}, \mathcal{R} = \begin{bmatrix} O_3 & O_3 \\ O_3 & D \end{bmatrix}, \tag{8}$$

the external connection matrices

$$g_\tau = \begin{bmatrix} O_3 \\ I_3 \end{bmatrix}, \quad g_b(q_3) = \begin{bmatrix} O_3 \\ J^T(q_3) \end{bmatrix}, \tag{9}$$

and the Hamiltonian function given by

$$H = \frac{1}{2}\mathbf{p}^T M^{-1}\mathbf{p}. \tag{10}$$

Note that the Hamiltonian function contains only a *kinetic* energy term, associated with the momentum variable. A *potential* energy, artificially added by the controller will play a key role to stabilize the ship in the desired position. From (2), we observe that the passive output for the system (7) is the velocity vector which does not correspond to the actual output of the system, the position and the orientation. This is an important consideration for the control design, especially for the dynamic IDA-PBC control design in Section 5.

## 4. Static IDA-PBC Controllers

A family of static feedback controllers via IDA-PBC can be obtained for the model described by the equations (7)-(10). As has been described in Section 2, let us consider the following desired closed loop dynamics,

$$\dot{\mathbf{x}} = (\mathfrak{J}_d - \mathcal{R}_d)\partial H_d = \begin{bmatrix} 0_3 & J(q_3) \\ -J^T(q_3) & R_p \end{bmatrix} \partial H_d, \tag{11}$$

where $R_p$ is a symmetric positive semi-definite matrix and the desired energy function is

$$H_d(\mathbf{p}, \mathbf{q}) = \Psi(\mathbf{q}) + \tfrac{1}{2}\mathbf{p}^T M^{-1}\mathbf{p}, \tag{12}$$

with the scalar function $\Psi(\mathbf{q})$ such that $\partial_q \Psi|_{q=0} = 0$ and $\partial_q^2 \Psi|_{q=0} > 0$. Following the details described in Section 2, we get the following static IDA-PBC controller

$$\boldsymbol{\tau} = -J^T(q_3)\,\partial_q \Psi - K_D J^T(q_3)\dot{\mathbf{q}} - J^T(q_3)\mathbf{b}, \tag{13}$$

with $K_D = R_p - D$, this can be seen as a nonlinear output feedback PD controller with a feed-forward term $-J^T(q_3)\mathbf{b}$. With two different formulations of the energy function $\Psi$,

$$\Psi_{quad} = \frac{1}{2}\mathbf{q}^T K \mathbf{q} \;\text{ and, }\; \Psi_{trig} = \frac{1}{2}[q_1\ q_2\ (1-\cos q_3)]^T C \begin{bmatrix} q_1 \\ q_2 \\ 1-\cos q_3 \end{bmatrix},$$

we get the following controllers

$$\boldsymbol{\tau}_{sq} = -J^T(q_3)(K\mathbf{q} + \mathbf{b}) - K_D J^T(q_3)\dot{\mathbf{q}}, \tag{14}$$

$$\boldsymbol{\tau}_{st} = -J^T(q_3)C\begin{bmatrix} q_1 \\ q_2 \\ \sin q_3 \end{bmatrix} - K_D J^T(q_3)\dot{\mathbf{q}} - J^T(q_3)\mathbf{b}, \tag{15}$$

where the matrices K and C are positive definite gain matrices.

The role of the energy function in this study is that of a Lyapunov function. From the Lyapunov stability theory, we know that the stability properties of a dynamical system and the minima of the Lyapunov funtion have a close connection. In the sequel, we explain how this connection be exploited to improve the performance of the controller in our study. In particular, we consider a quadratic and a trigonometric energy shaping. Figure 3 gives an idea of the shape of the level surfaces corresponding to each energy shaping.

The main motivation for the trigonometric energy shaping is that for certain applications where there are no external constraints (for instance, links with external objetcs), stabilization in $q_3 = 0$ or $q_3 = 2\pi$ is exactly the same. Figure 4, shows a possible scenario where the path for stabilization in $q_3 = 2\pi$ is shorter than the path for stabilization in $q_3 = 0$. We will refer the controller described by (14) as static quadratic and the one described by (15) as static trigonometric controllers, respectively.

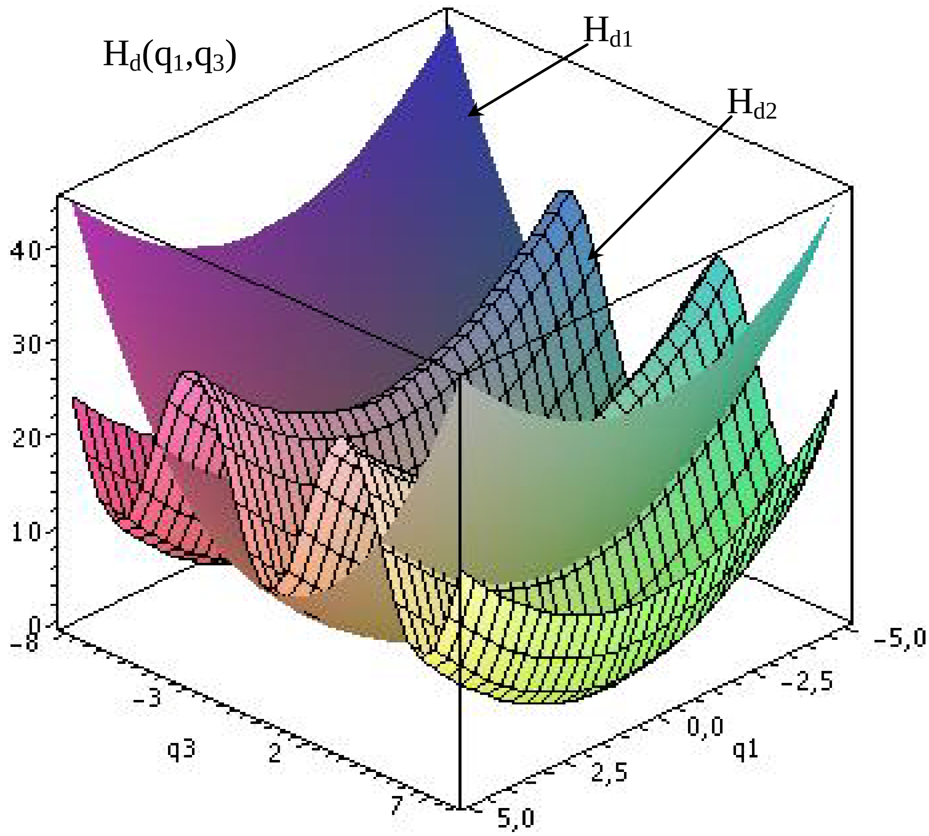


*Figure 3. A comparison between the quadratic and trigonometric energy functions in $q_1$ and $q_3$.*

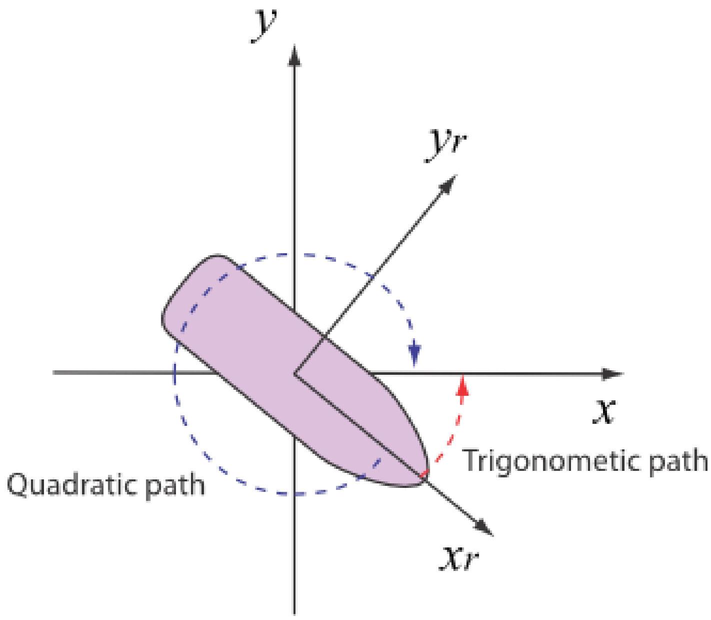


*Figure 4. Advantage of the trigonometric controller over the static controller.*

The stability properties of the proposed controllers are summarized in the following Proposition.

**Proposition 4.1:** *Consider the dynamical system (7) in a closed loop with the control laws (14) or (15), where the origin* $\mathbf{q} = 0$ *is a minimum of* $\Psi(\mathbf{q})$, *and the bias vector* $\mathbf{b}$ *and the matrix D are known. Then, the desired regulation point* $(\mathbf{q}^*, \mathbf{p}^*) = (0,0)$ *is (locally) asymptotically stable.*

## 5. Dynamic IDA-PBC Controllers

The static controllers described in the previous Section have nice stabilizing properties for the nominal case (when there is no disturbance or the disturbance is known perfectly). For the non-nominal case, the static controllers do not give the desired stabilizing properties. As has been mentioned in Section 2, this discrepancy can be removed by A dynamic extension for a non passive output, maintaining the port- Hamiltonian structure, is possible by means of a change of coordinates. Following the idea in [1], we introduce a new state variable, $\mathbf{z_e} \in \mathbb{R}^3$, which is used to enforce the equilibrium point of the closed loop system to the desired one, and a change of variables $\mathbf{z} = \mathrm{f}(\mathbf{q}, \mathbf{p}, \mathbf{z_e})$. We define the target system as

$$\begin{bmatrix} \dot{\mathbf{q}} \\ \dot{\mathbf{z}} \\ \dot{\mathbf{z}}_e \end{bmatrix} = \begin{bmatrix} 0_3 & J(q_3) & J(q_3) \\ -J^T(q_3) & -R_z & 0_3 \\ -J^T(q_3) & 0_3 & 0_3 \end{bmatrix} \begin{bmatrix} \partial_q H_{de} \\ \partial_z H_{de} \\ \partial_{z_e} H_{de} \end{bmatrix}, \qquad (16)$$

where $R_z = R_z^T \geq 0$, is a $3 \times 3$ matrix. The desired Hamiltonian function is defined as

$$H_{de}(\mathbf{q}, \mathbf{z}, \mathbf{z_e}) = \Psi(\mathbf{q}) + \frac{1}{2}\, \mathbf{z}^T M^{-1} \mathbf{z} + \frac{1}{2}\, \mathbf{z}_e^T K_e \mathbf{z}_e, \qquad (17)$$

where $K_e$ is a positive definite gain matrix. As for the static case, depending on the formulation of $\Psi(\mathbf{q})$, we get the following two control laws

$$\tau_{dq} = -K_p J^T(q_3)K\mathbf{q} + K_I\mathbf{z}_e - K_D J^T(q_3)\dot{\mathbf{q}} - J^T(q_3)\mathbf{b}, \quad (18)$$
$$\dot{\mathbf{z}}_e = -J^T(q_3)K\mathbf{q}, \quad (19)$$

$$\tau_{dt} = -K_p J^T(q_3)C\begin{bmatrix} q_1 \\ q_2 \\ \sin q_3 \end{bmatrix} + K_I\mathbf{z}_e - K_D J^T(q_3)\dot{\mathbf{q}} - J^T(q_3)\mathbf{b}, \quad (20)$$
$$\dot{\mathbf{z}}_e = -J^T(q_3)C\begin{bmatrix} q_1 \\ q_2 \\ \sin q_3 \end{bmatrix}, \quad (21)$$

where $K$ and $C$ are the positive definite gain matrices and other gain matrices are defined as

$$K_p := MK_e + I_3, \quad (22)$$
$$K_D := R_z - D, \quad (23)$$
$$K_I := K_e. \quad (24)$$

We will refer to the controller described by (18)-(19) as the dynamic quadratic controller and the one described by (20)-(21) as dynamic trigonometric controller. The stability properties of these controllers are summarized in the following proposition.

**Proposition 5.1:** *Assume that q is measurable, and that the disturbances vector b and the matrices M and D are known. If* $K_e = \mathrm{diag}\{k_{e1}, k_{e2}, k_{e3}\} > 0$*, and* $\Psi(\mathbf{q})$ *has a (local) minimum at the origin,* $\mathbf{q} = 0$*, then the system (7) in a closed loop with both the controllers defined by (18)-(19) and (20)-(21), is (locally) asymptotically stable at the point* $(\mathbf{q}, \mathbf{z}, \mathbf{z}_e) = (0,0,0)$.

*Furthermore, if* $\mathbf{q} = 0$ *is a global minimum of* $\Psi(q)$*, then the origin of (7) is globally asymptotically stable.*

## 6. <u>Simulations</u>

In order to test the performance of the designed controllers we performed some numerical simulations. For this validation we used the data of a supply ship from [3]. The (Bis-scaled non-dimensional [2]) matrices M and D are given by

$$M = \begin{bmatrix} 1.1274 & 0 & 0 \\ 0 & 1.8902 & -0.0744 \\ 0 & -0.0744 & 0.1278 \end{bmatrix}, D = \begin{bmatrix} 0.0358 & 0 & 0 \\ 0 & 0.1183 & -0.0124 \\ 0 & -0.0124 & 0.0308 \end{bmatrix},$$

The bias vector has been set to $\mathbf{b} = [0.05 \quad 0.05 \quad 0.01]^T$ . For all simulation we considered that the initial conditions of the ship are $(q_1, q_2) = (-10, -10)$, and the heading angle $q_3 = 4$ rad, and the desired stabilization position is the origin. Precisely, the starting heading angle is set greater than $\pi$ to show the ability of the so-called trigonometric controller to stabilize to the closer minimum, in this case $2\pi$.

### 6.1 Simulation results for the static controller

Here, we present the simulation results for the static (quadratic and trigonometric) control laws (14) and (15), respectively. The gain matrices we used are $K = C = \mathrm{diag}\{0.05, 0.05, 0.01\}$ and $R_p = \mathrm{diag}\{0.75, 0.75, 0.1\}$. Precisely, for this system, we enlarged the damping (about one order of magnitude in the first and third components), to improve the performance.

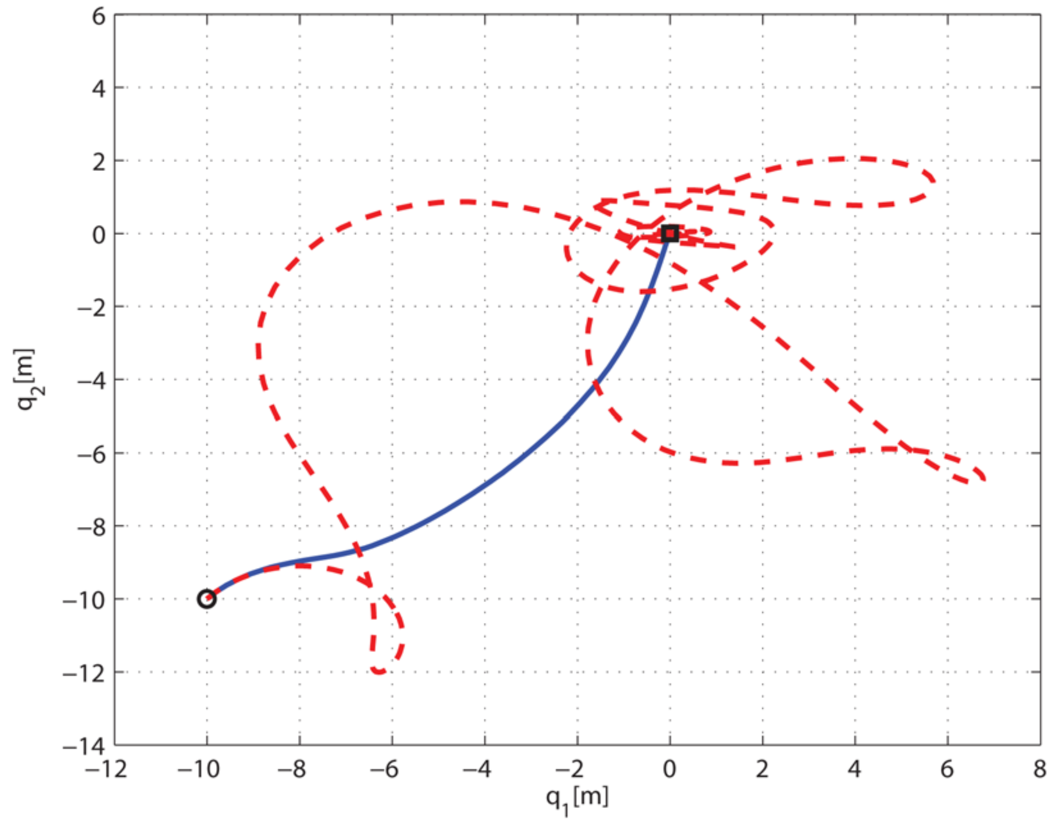


*Figure 5 Simulation results: ship position trajectories in the $q_1q_2$ -plane, for the static quadratic controller with extra damping (solid line) and the original damping coefficient (dashed line).*

In Figure 5 we show the trajectories of the same static controller (the quadratic case) with extra dissipation, setting Rp to the values proposed before, and keeping the original damping, Rp = D. This comparison justifies the use of the extra damping to obtain more suitable paths.

Figure 6 shows the trajectories of the position coordinates $q_1$ and $q_2$, and the heading angle, $q_3$, of the quadratic and the trigonometric versions. Both controllers stabilize the ship at the desired position and angle.

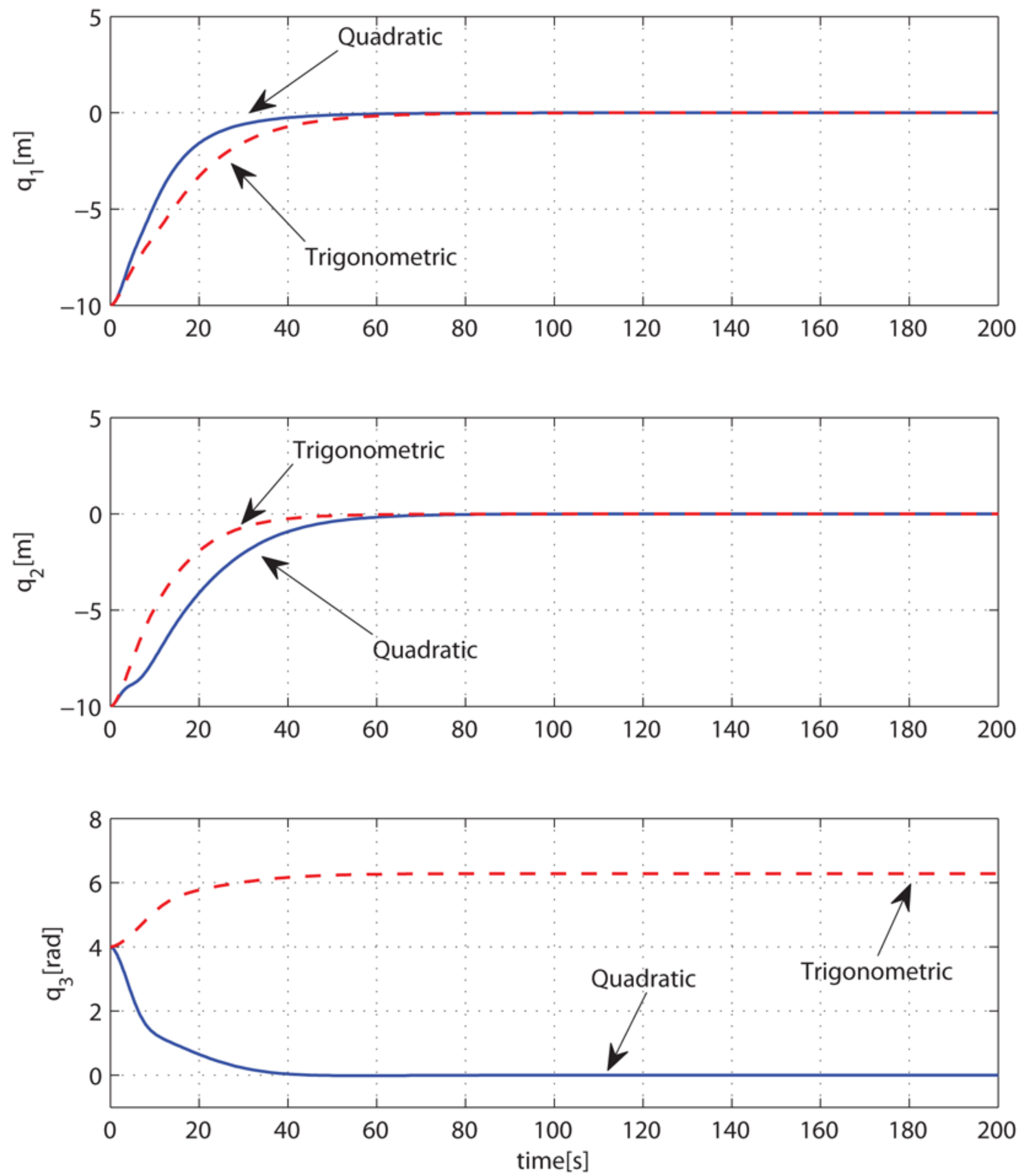


*Figure 6. Simulation results: ship position coordinates $q_1$ and $q_2$, and heading angle, $q_3$, for the static quadratic (solid line) and trigonometric (dashed line) controllers.*

The notable point is the difference in the orientation profiles. While the quadratic controller stabilizes the heading angle at $q_3 = 0$, the trigonometric controller does so at $q_3 = 2\pi$.

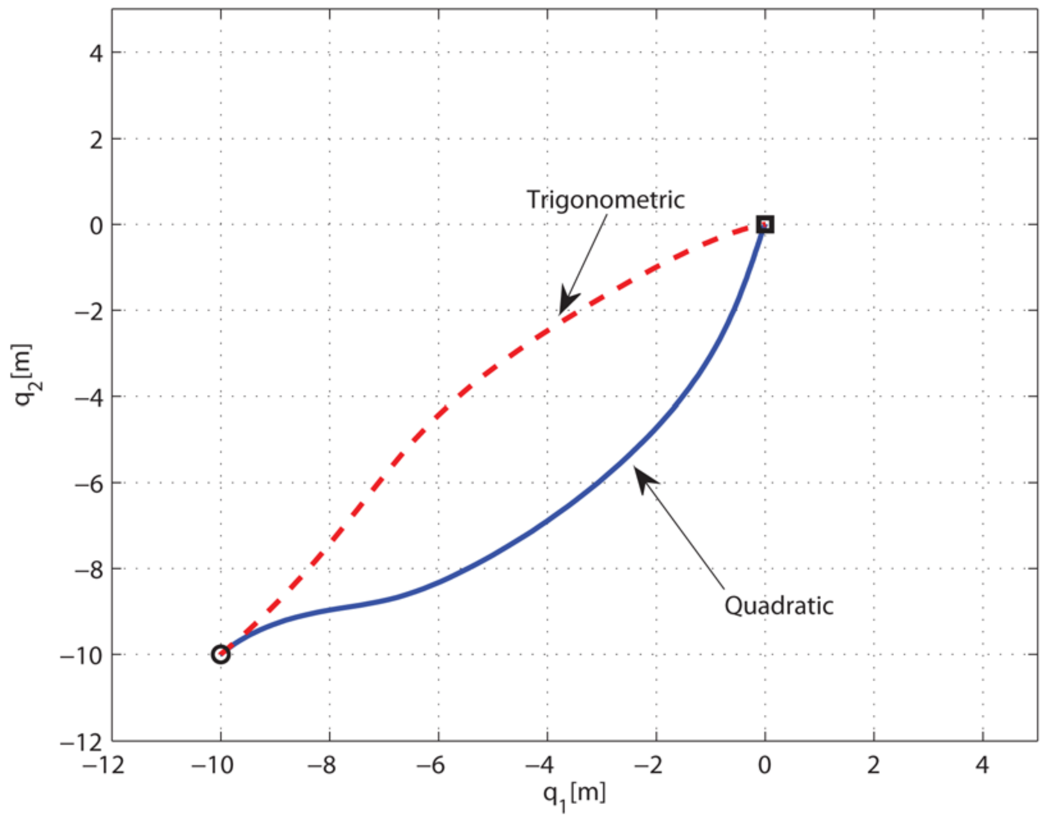


*Figure 7. Simulation results: ship position trajectories in the $q_1 q_2$-plane, for the static quadratic (solid line) and trigonometric (dashed line) controllers.*

Figure 7 compares the trajectories in the $q_1 q_2$-plane for two versions of the static controller. In both cases, the performance is similar but, even the controllers for the $q_1$ and $q_2$ coordinates are the same (with the same gain values), the trajectories take different paths. This fact is associated with the different heading angle trajectories.

### 6.2 Simulation results in presence of disturbances

Here, we present the simulation results in presence of disturbances. The key point is to show that the dynamic controllers proposed are able to reject unknown terms. For this scenario we considered that the disturbance due to the bias term, **b**, is not available. Consequently, the feed-forward term, $J^T(q_3)\,\mathbf{b}$, is removed in all the tested controllers. The gain matrices for the static controllers, (14) and (15), are the same as in the previous subsection. The corresponding gain matrices used for both the (quadratic and trigonometric) dynamic controllers, (18)-(19) and (20)-(21), are $R_z = \mathrm{diag}\{0.75, 0.75, 0.4\}$, $K = C = \mathrm{diag}\{0.05, 0.05, 0.025\}$ and $K_e = \mathrm{diag}\{0.01, 0.01, 0.015\}$.

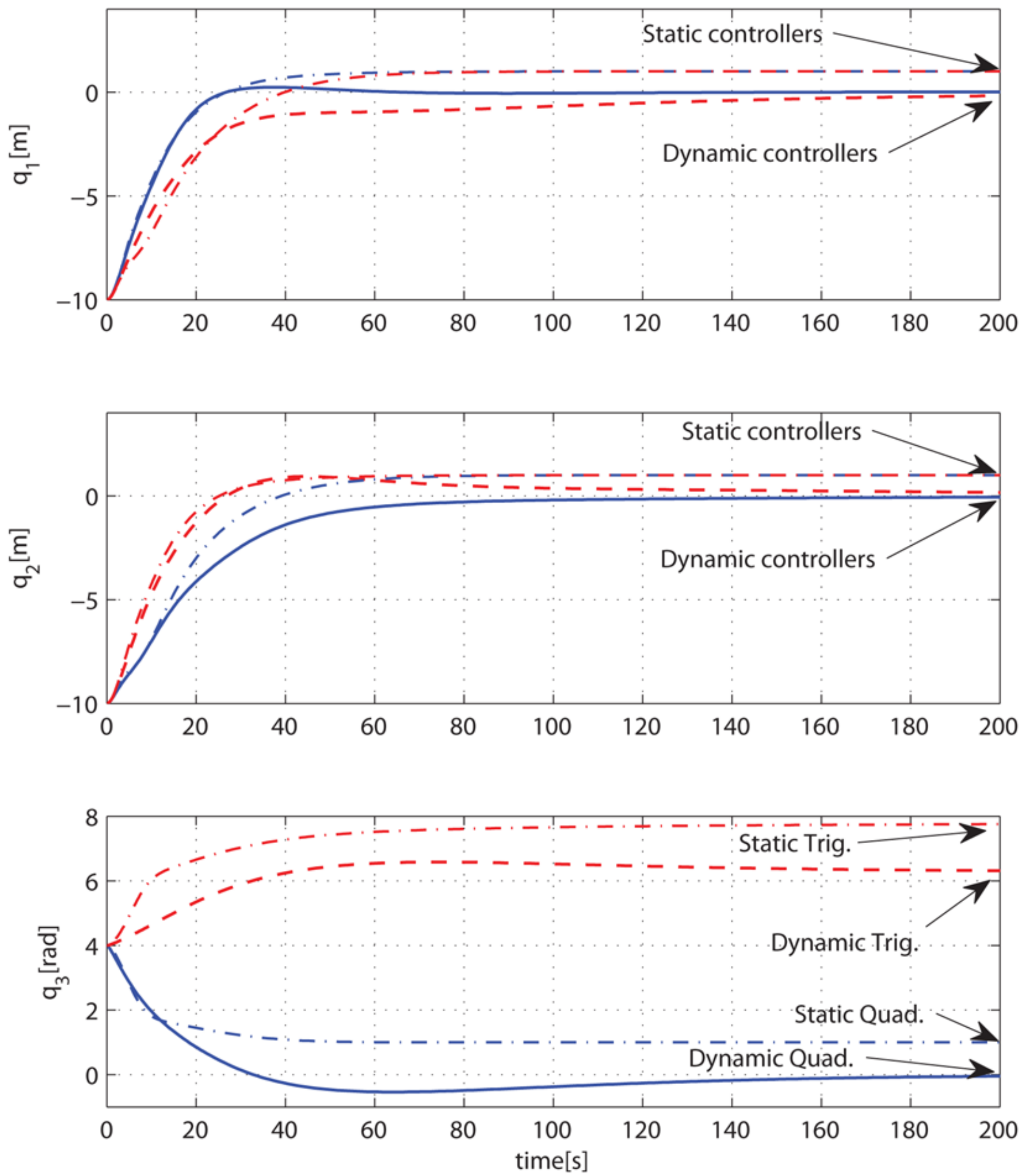


*Figure 8. Simulation results: ship position coordinates, $q_1$ and $q_2$, and heading angle, $q_3$, for the static and dynamic (quadratic and trigonometric) controllers.*

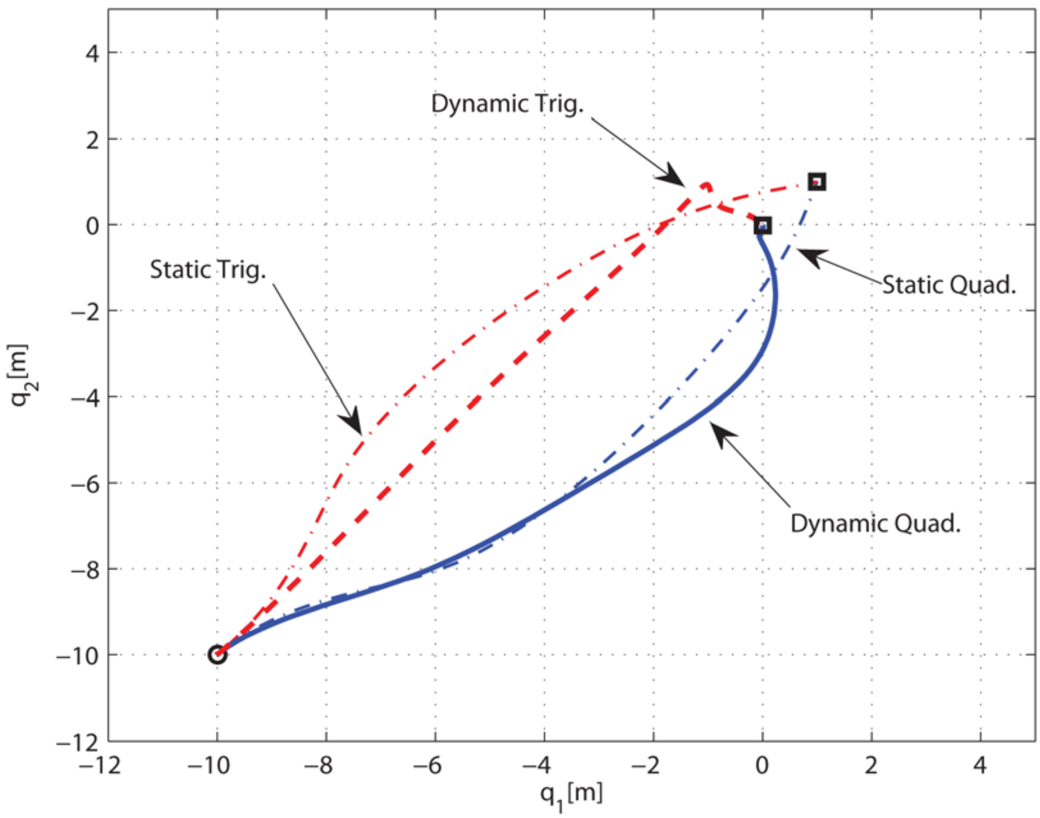


*Figure 9. Simulation results: ship position trajectories in the $q_1$, $q_2$ plane, for the static and dynamic (quadratic and trigonometric) controllers.*

In Figure 8, **q** trajectories for the four controllers are plotted. Clearly the dynamic controllers steer the ship to the desired equilibrium position while the static controller fails to do so and have some steady state error. This difference between the performance of the static and the dynamic controllers can also be seen from the respective trajectory profiles, see Figure 9.

## 7. Conclusions

A passivity-based approach called IDA-PBC is used to obtain a set of controllers for the dynamic positioning of a ship. This methodology is based on the port-Hamiltonian description which gives a physical interpretation of the dynamical systems. Under this point of view, the controller design problem is addressed as to shape the energy function of the closed loop system. After a general formulation we propose two different controllers: first with a quadratic energy function and second, inspired by the physics of a pendulum, with a trigonometric energy function. Also, the presence of disturbances is studied and it turns out that the obtained static control laws do not stabilize the system at the desired position. This discrepancy is the starting point for a second set of controllers which consists of a dynamic extension of the system which provides stability at the desired regulation point, also in presence of disturbances. Simulations are done to validate and compare the performance of the controllers designed.

It is worth to mention that the obtained control laws, with a general form of state feedback, can be easily converted to output feedback algorithms that only require the position measurement.

Furthermore, they exhibit a simple structure that can be interpreted as non-linear version of PID controllers.

Future work can be oriented in to determine other energy functions, $\Psi(\mathbf{q})$, to improve the performance, as well as to consider the optimization of the resulting path. Further analysis depending on the nature of the disturbance vector (including the wave frequency and wind models) are possible. Also, this work could be a starting point for a new design, using the port-Hamiltonian perspective, of the complete motion-control system (controller and observer) for a DP problem.

**References**

[1] A. Donaire and S. Junco. On the addition of integral action to port-controlled Hamiltonian systems, *Automatica*, 45: 1910-1916, 2009.
[2] T. I. Fossen. *Guidance and Control of Ocean Vehicles*, John Willey & Sons, 1994.
[3] T. I. Fossen and Å. Grøvlen. Nonlinear output feedback control of dynamically positioned ships using vectorial observer backstepping. *IEEE Transactions on Control Systems Technology*, 6:121-128, 1998.
[4] T. I. Fossen and J. P. Strand. Passive Nonlinear observer design for ships using Lyapunov methods: Full scale experiments. *Automatica*, 35:3-16, 1999.
[5] T. I. Fossen and T. Perez. Kalman filtering for positioning and heading control of ships and offshore rigs. *IEEE Control Systems Magazine*, 29: 32-46, 2009.
[6] P. T-K. Fung and M. J. Grimble. Dynamic ship positioning using a self tuning Kalman filter. *IEEE Transactions on Automatic Control*, 28: 339-350, 1983.
[7] M. J. Grimble, R. J. Patton, and D. S. Wise. Use of Kalman filtering techniques in dynamic ship positioning systems. *Proceedings of the IEE*, Part D, 127:93:102, 1980.
[8] H. Katayama. Nonlinear sampled-data stabilization of dynamically positioned ships. *IEEE Transactions on Control Systems Technology*, 18:463-468, 2010.
[9] A. Loria, T. I. Fossen, and E. Panteley. A separation principle for dynamic positioning of ships: theoretical and experimental results. *IEEE Transactions on Control Systems Technology*, 8:332-343, 2000.
[10] S. Muhammad. Dynamic Positioning of ships using Thrusters and Controllers. PhD Thesis. To appear in 2011.
[11] R. Ortega and E. García-Canseco. Interconnection and damping assignment passivity-based control: A survey. *European Journal of Control*, 10:432-450, 2004.
[12] R. Ortega, A. van der Schaft, B. Maschke, and G. Escobar. Interconnection and damping assignment passivity-based control of port-controlled Hamiltonian systems. *Automatica*, 38:585-596, 2002.
[13] J. G. Snijders. *Wave filtering and thrusters allocation for dynamic positioned ships*. M. Sc. Thesis, Mathematical Systems Theory Group, DIAM, EWI Faculty, TU Delft, the Netherlands, 2005.
[14] A. J. Sørensen, S. I. Sagatun, and T. I. Fossen. Design of dynamic positioning system using model-based control. *Control Engineering Practice*, 4:359-368, 1996.
[15] A. J. van der Schaft. *$L_2$ gain and passivity techniques in nonlinear control*, Springer, 2000.