Design and Control

Exponentially Stable Underactuated Dynamic Positioning of Marine Vehicle

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Research Sponsored by the Office of Naval Research

Electric Ship Research & Development Consortium

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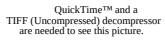
are needed to see this picture.

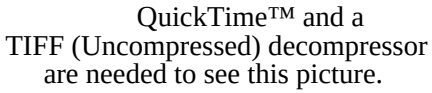
Outline

- Motivations: Why we're interested
- Technical Challenges: Why it's hard
- State Feedback Controller: Partial solution
- Manifold Convergence Controller: The rest
- Simulation Results: Perfection
- Experimental Results: Reality
- Conclusions

General Dynamic Positioning

- Requirements:
  - Many thrusters
  - Thrust allocation algorithm





Images: dosecc.org

Underactuated DP

- Fully actuated ship: can generate an acceleration in any direction (surge, sway, yaw)
  - Need at least three independent inputs: 1 azimuth thruster + 1 bow thruster for example
- Underactuated ship: only two inputs
  - 1 azimuth thruster
  - Fixed shaft + rudder
  - Outboard engine?
  - Small craft!

Applications

- Stationkeeping autopilots for fishing vessels and rescue boats
- DP for DP vessels damaged beyond redundancy
- ASV/AUV positioning

Challenge of Underactuation

- Can't reject most disturbances with only two inputs!
  - Fore-aft errors OK, transverse/rotation errors not
- General approaches to underactuated control:
  - Trajectory planning
  - Energy Methods
  - Switched Control
- Planning = High-order Nonlinear Optimization:
  - Computationally intensive
  - Generally open-loop

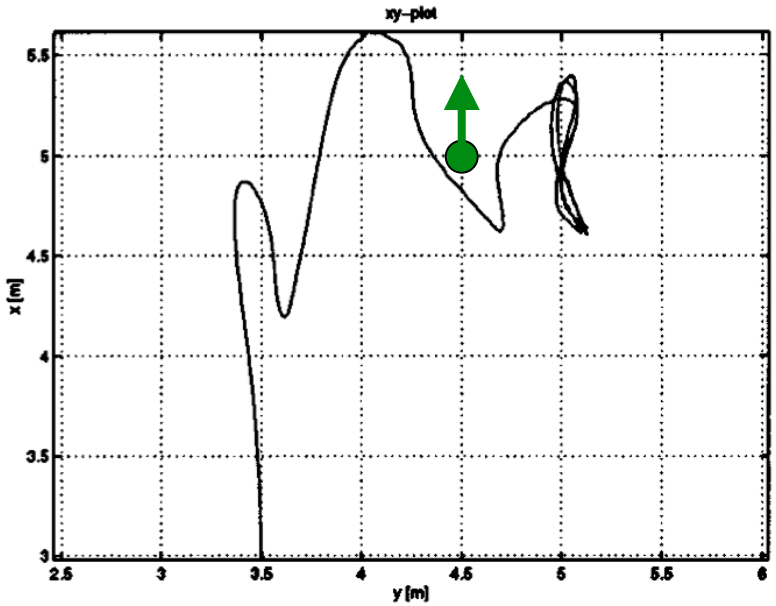
Mathematically…

$$\dot{\nu} = \mathbf{a}(\nu)\,\nu + \mathbf{b}\,\tau$$

accel 3x1 | velocity 3x1 | input 2x1

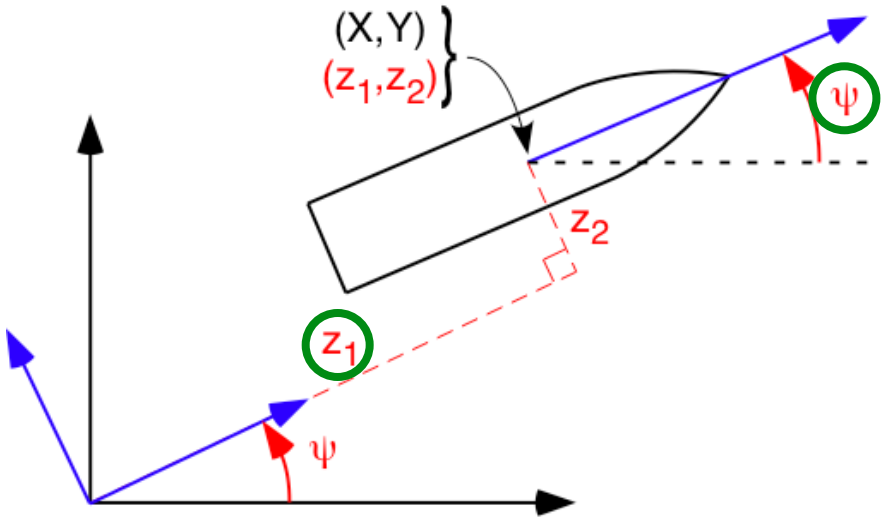
- Matrix b is not full rank, so underactuated
- But time-varying periodic control laws can work
- Parallel parking a car

- Pettersen 1996:
- Experiments 2000:



Well what CAN we do?

- With 2 inputs, we can potentially regulate 2 outputs with state feedback
- Linear Quadratic Regulator (LQR) or other…



$$
\begin{aligned}
z_1 &= X\cos\psi + Y\sin\psi \\
z_2 &= -X\sin\psi + Y\cos\psi \\
z_3 &= \psi
\end{aligned}
\qquad\qquad
\begin{aligned}
\dot{z}_1 &= u + z_2 r \\
\dot{z}_2 &= v - z_1 r
\end{aligned}
$$

Linear State Feedback

- State transition (linear damping, nonlinear kinematics):

$$\mathbf{x} = [u, v, r, z_1, z_2, \psi]^T \equiv \left[\boldsymbol{\nu}^T, \mathbf{z}^T\right]^T$$

$$\begin{aligned}\dot{\mathbf{x}} &= \begin{bmatrix} \mathbf{a}(\boldsymbol{\nu})\boldsymbol{\nu} \\ \dot{\mathbf{z}} \end{bmatrix} + \begin{bmatrix} \mathbf{b} \\ \mathbf{0} \end{bmatrix} \boldsymbol{\tau} \\ &\equiv \mathbf{f}(\mathbf{x}) + \mathbf{B}\boldsymbol{\tau}\end{aligned} \qquad \mathbf{f}(\mathbf{x}) = \begin{bmatrix} a_{11}u + \frac{m_{22}}{m_{11}}vr \\ a_{22}v + a_{23}r - \frac{m_{11}}{m_{22}}ur \\ \frac{m_{11}-m_{22}}{m_{33}}uv + a_{32}v + a_{33}r \\ u + z_2 r \\ v - z_1 r \\ r \end{bmatrix}$$

every state except z_2 (transverse error)

- LQR controller: $J = \int_0^\infty \mathbf{x}^T \mathbf{Q}_5 \mathbf{x} + \boldsymbol{\tau}^T \mathbf{R} \boldsymbol{\tau} dt$

State Feedback Control Law

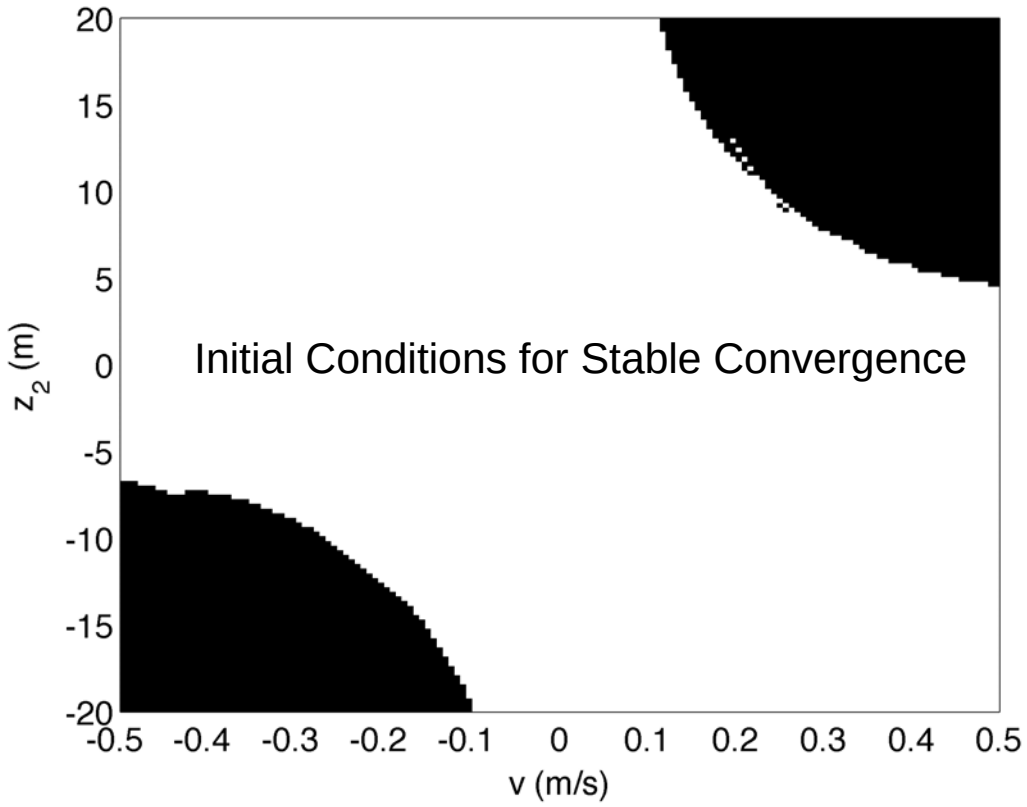
$$\begin{aligned}\boldsymbol{\tau} &= -\mathbf{R}^{-1}\mathbf{B}^T\mathbf{C}_5^T\mathbf{P}_5\mathbf{C}_5\mathbf{x} \\ &\equiv -\mathbf{K}\mathbf{x}\end{aligned}$$

Closed Loop Dynamics

$$\begin{aligned}\dot{\mathbf{x}} &= (\mathbf{f}_\mathbf{x} - \mathbf{B}\mathbf{K})\mathbf{x} \\ &= \mathbf{f}_\mathbf{x}^{\mathbf{cl}}\mathbf{x}\end{aligned}$$

Region of Convergence

- Strong z_2v dependence, weak u dependence
- Region of convergence large in this example (LOA = 3.71 m)

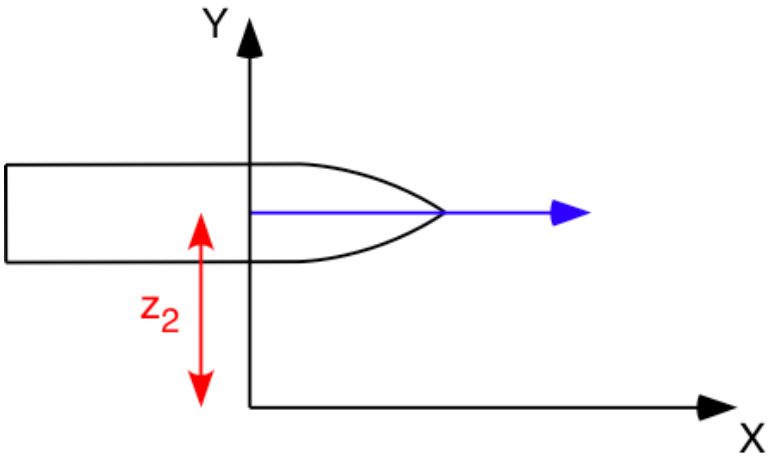


Great… now what?

- The LQR (or other) state feedback controller will drive all states except z_2 to zero

$$\begin{matrix} & u & v & r & z_1 & z_2 & \psi \\ \mathbf{x}(\infty) = [& 0 & 0 & 0 & 0 & z_2(\infty) & 0 \ ]^T \end{matrix}$$

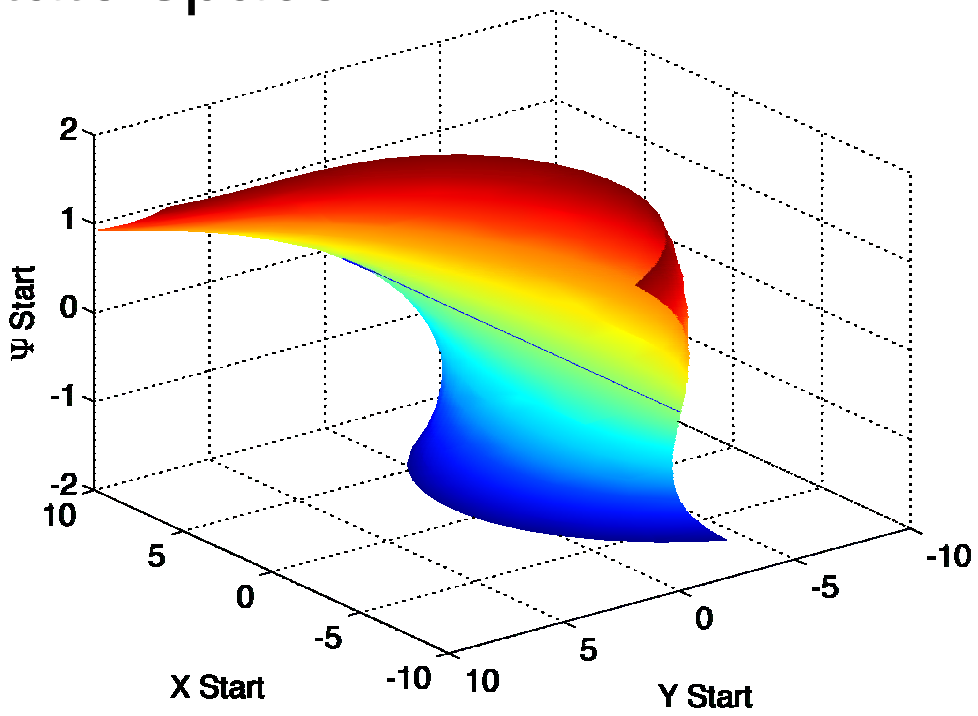
- Vessel ends up on the global Y axis pointing along the X axis



  - With a simple simulation we can predict $z_2(\infty)$ from any initial condition!

The Manifold

- For certain initial conditions the final (time infinity) state vector will be zero
- This set of initial conditions forms a manifold in the state space



Algorithm

1. Get to the manifold — HARD?
2. Drive along the manifold to the goal — EASY!
3. Get back to the manifold if you get pushed off

Intuition (behavior we want)

- For small disturbances, just use the state feedback controller and accept minor errors
- For larger disturbances, you need to reposition yourself… may need to back off from the goal for a moment (parallel parking analogy)

Manifold Convergence Controller

- Find a positive definite function of some states of interest; if its derivative with respect to time is negative definite then those states will go to zero

$$V = \frac{1}{2} {z_{2\infty}}^2$$

predicted final lateral offset

$$\begin{aligned} \dot{V} &= \frac{\partial V}{\partial \mathbf{x}} \frac{d\mathbf{x}}{dt} \\ &= \frac{\partial V}{\partial \mathbf{x}} \left((\mathbf{f_x} - \mathbf{BK})\,\mathbf{x} + \mathbf{B}\,\boldsymbol{\tau_l} \right) \\ &= z_{2\infty} \frac{\partial z_{2\infty}}{\partial \mathbf{x}} (\mathbf{f_x} - \mathbf{BK})\,\mathbf{x} + z_{2\infty} \frac{\partial z_{2\infty}}{\partial \mathbf{x}} \mathbf{B}\,\boldsymbol{\tau_l} = c_v \end{aligned}$$

control input

negative definite

$-V/T_V$

Manifold Convergence Controller (MCC)

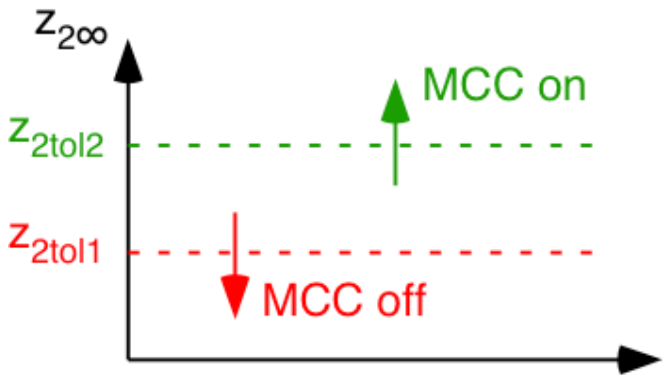
$$\boldsymbol{\tau_l} = -pinv\left(\frac{\partial z_{2\infty}}{\partial \mathbf{x}}\mathbf{B}\right)\left(\frac{1}{2T_V} z_{2\infty} + \frac{\partial z_{2\infty}}{\partial \mathbf{x}}(\mathbf{f_x} - \mathbf{BK})\mathbf{x}\right)$$

numerical derivative

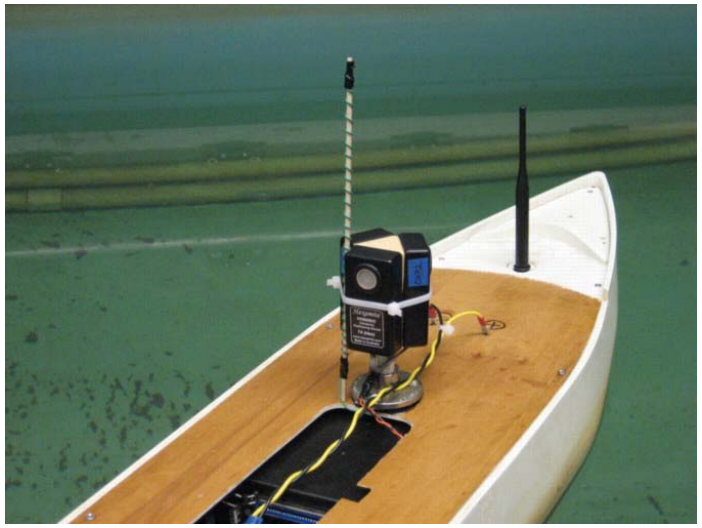
state feedback!

Wait a sec!

- We're controlling a future state with current inputs… that could be problematic!
- … and it is! The MCC gets to the manifold asymptotically, but not the goal.
- When within some tolerance around the manifold, switch to the state feedback controller which drives toward the goal.
- Use a switching boundary with hysteresis:
  - Turn off MCC when inside a certain radius, turn it back on if you get outside a larger radius



Simulation of Icebreaker Model



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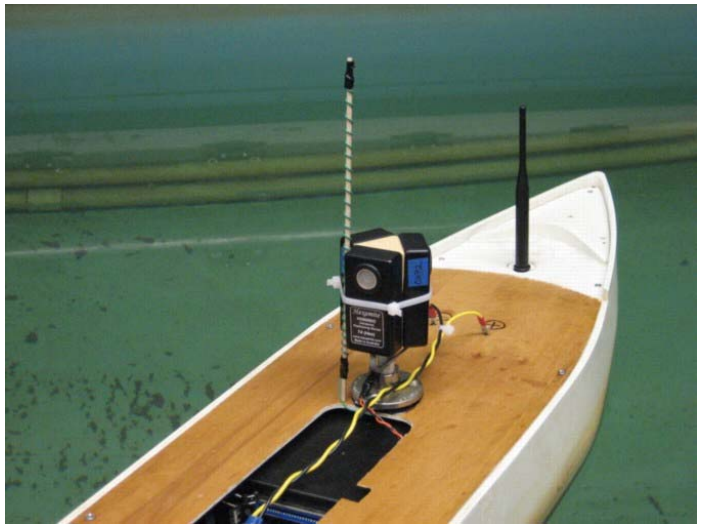
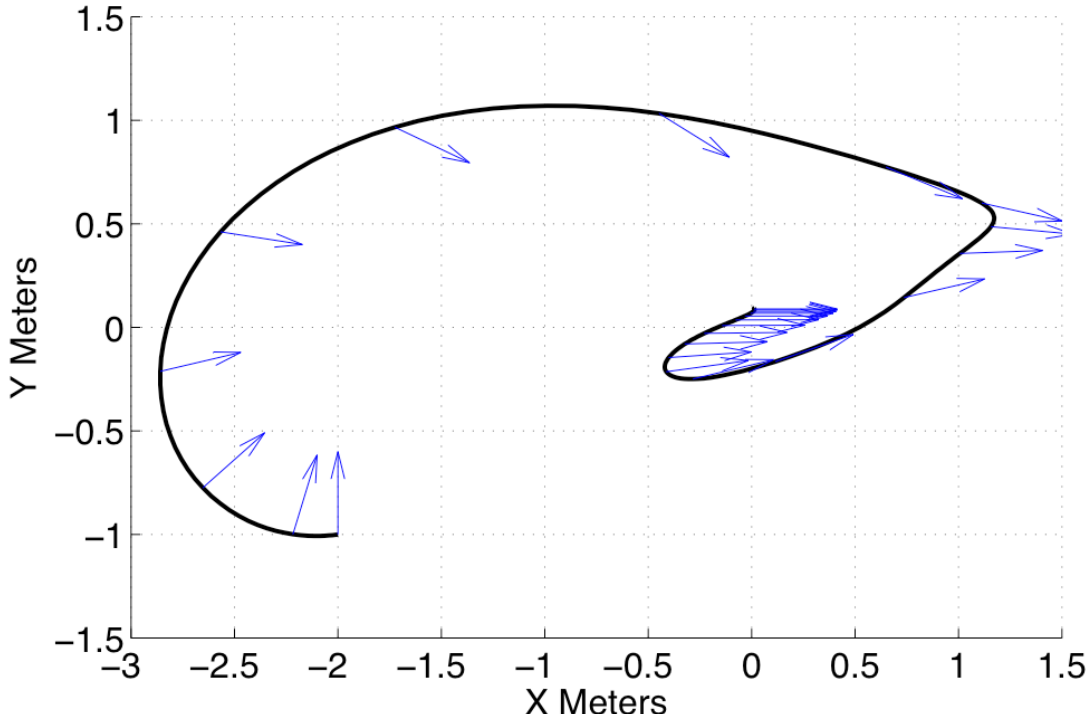
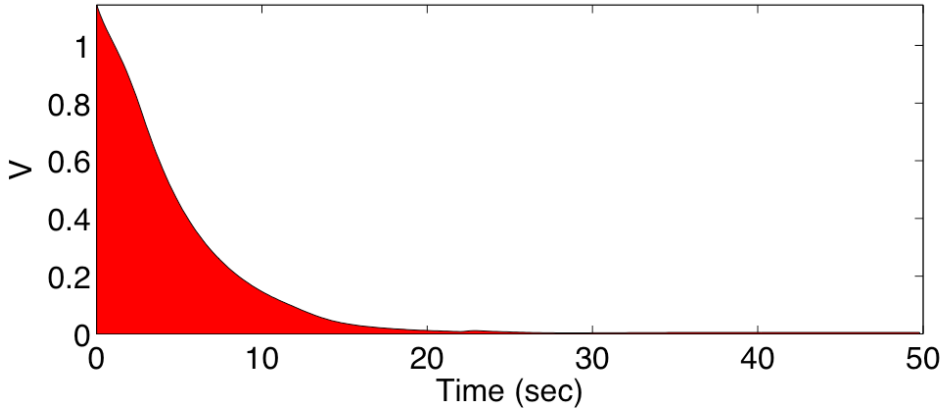
Simulation Results







Correct for Modeling Error (experiment)

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On the Charles River (experiment)

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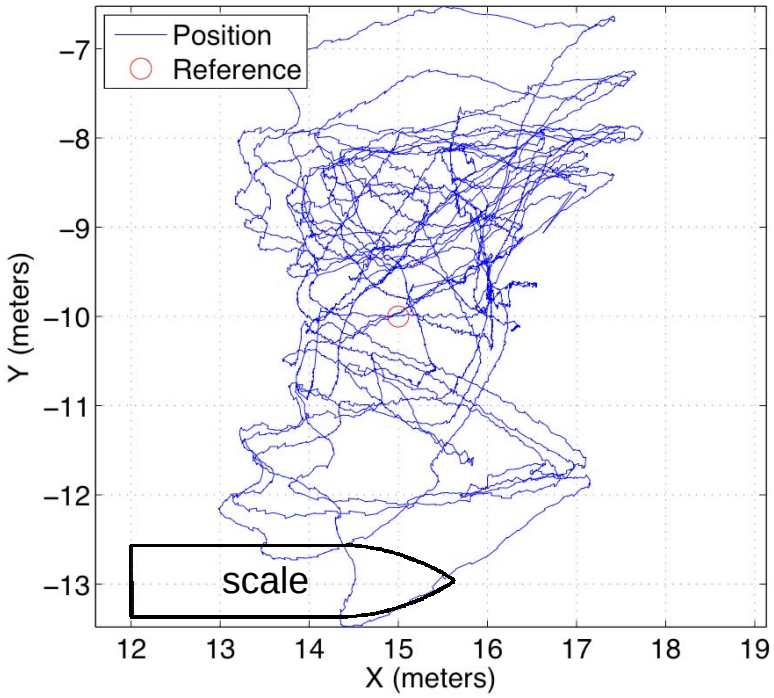
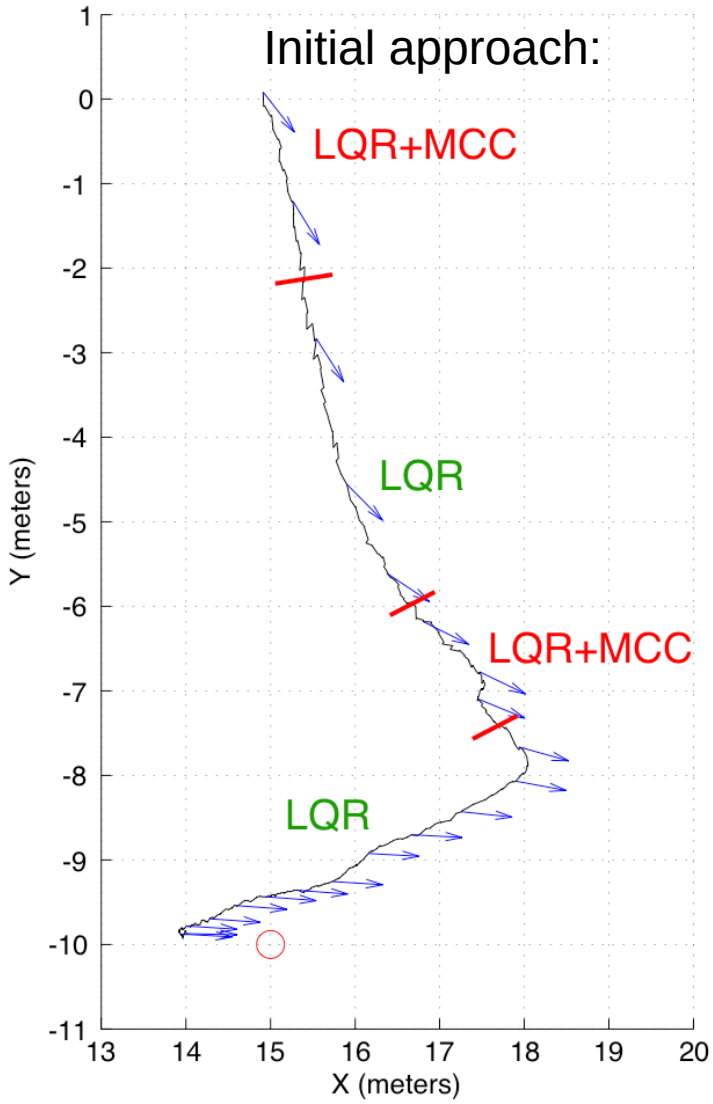
On the Charles River (experiment)





Conclusions

- Underactuated DP is possible if the vessel is allowed to reposition itself when errors get too large
  - Minimize future errors with current actions
  - Exponentially stable switching control law
  - Intuitive results
- Issues:
  - Need a very good model of the vessel and, if possible, environmental conditions (wind & current)
  - Errors not driven exactly to zero

What's Next?

- Use a better (nonlinear) model in the forward simulations
- Include known environmental forces
- Extend to 5 DOF with 3 input channels (AUV with propeller, rudder & dive planes)

Questions?