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Automatic Heading Control for Dynamic Positioning in Ice

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Abstract

Offshore activities in ice-covered waters are gaining increased attention nowadays. Such operations as crew change, lifting, installation, drilling, etc. may require keeping the vessel on a fixed location during long time periods. The use of dynamic positioning systems appears therefore to be an attractive solution, being much more flexible than mooring operations. However, the ice environment is significantly different from open water conditions. As known from several full-scale experiments, systems developed for open water purpose do not answer all ice challenges. Nevertheless, recent R&D projects have demonstrated the feasibility of DP in ice. During the DYPIC project, a European collaborative program, large amount of ice model basin tests have been performed at the Hamburg Ship Model Basin. Those tests have not only spotlighted stationkeeping possibilities under certain challenging conditions, but have also brought out several important aspects of ice forces on the hull. In this paper, the DYPIC outcomes are used to build an automatic heading control system for a DP vessel in order to improve its stationkeeping abilities in ice. The system with automatic heading is then compared in a numerical simulation framework to a system with fixed heading. The results show that in selected scenarios the DP system with automatic heading control outperforms clearly the one with fixed heading control. Specifically, the ice loads and the power consumption are reduced considerably.

1. Introduction

Arctic areas are becoming increasingly attractive for a wide spectrum of various industries. As underlined e.g. by Naseri and Barabady in [1] this interest emerges from several factors, not only because of the considerable resources located there (16% of worlds' undiscovered oil and 25% of worlds' undiscovered gas). Furthermore, the technological progress in all areas of the ship industry (thruster manufacturing, ship design, power management systems, etc.), coupled to the trending decrease of the Arctic ice cover, lead to consider these areas for possible offshore developments in the near future. Offshore operations are diverse and mostly complex, especially those involving geo-fixed positioning during prolonged time periods. As underlined in e.g. [2], Dynamic Positioning Systems (DPS) are considered as a possible solution for stationkeeping in ice. The DYPIC project [3] has indeed demonstrated the feasibility of such operations under challenging environmental conditions (Figure 1).

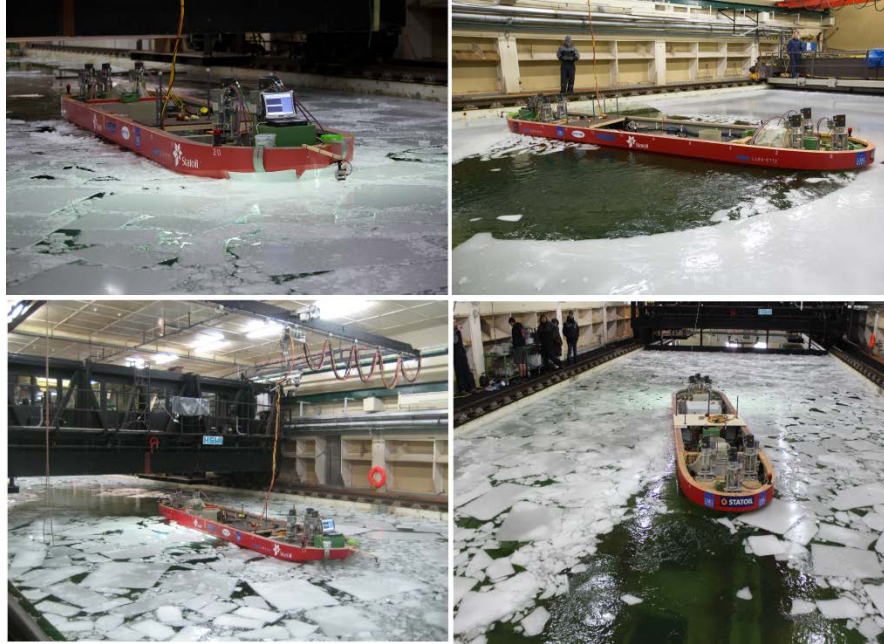


Figure 1. Ice basin tests at HSVA within the Dynamic Positioning in Ice (DYPIC) project - dypic.eu.

This paper is structured as follows: Section 2 reviews the challenges associated with dynamic positioning in ice and motivates the interest for automatic heading control. Section 3 is dedicated to the description of this control strategy and its structure. The automatic heading control scheme is then benchmarked in simulation against a fixed heading strategy, using a high-fidelity numerical model developed by the Norwegian University of Science and Technology (NTNU) presented in Section 4. The simulation scenarios and results are presented in Section 5 and discussed in Section 6. Finally, conclusions are drawn in Section 7.

2. The need for automatic heading control with DP in ice

Only a few full-scale dynamic positioning in ice experiences have been performed and are reported in the literature, see e.g. [3, 4]. Firstly, those experiments have demonstrated some deficiencies of open water DPS in ice conditions. The ice actions turned out to be very different from open water in a way that they are more severe, more complex and depend on a multitude of parameters [5]. Secondly, large discrepancies of the ice loads on the hull have been observed depending on the properties of the ice field [6] that could naturally lead to disparities in the stationkeeping performance. Finally, Ice Management (IM) has been reported as vital for stationkeeping operations [4]. IM is the sum of all different tasks and activities dedicated to reducing or avoiding ice actions [7], and it is usually used for reducing the severity of the ice field around the DP vessel, homogenizing ice actions on the hull and preventing undesired threats (such as ice ridges, icebergs, etc.).

In order to cope with the ice loads on the DP vessel, new control strategies have been developed and tested at the Hamburg Ship Model Basin (HSVA) in 2011 and 2012 within in the framework of DYPIC. The differences between open water-designed and ice-designed control laws have then been considered theoretically in [8], based on the outcomes of the R&D project.

DP abilities are naturally expected to be maximal when the ship is facing the ice drift with the bow or the stern. This has been validated by the first formulation of capability plots in ice proposed in [9] and confirmed for a thruster-assisted moored structure in [10]. The reader may note that both in [9] and [10] the calculations have been performed with the ship heading against the ice drift only. Nevertheless, the

strong sensitivity of the performances towards the ice drift oblique angle has been clearly demonstrated and it has been confirmed that having the vessel aligned with the ice direction is needed in order to reduce the level of ice loads on the hull [11]. This may also be beneficial with respect to the power consumption of the vessel, as will be demonstrated in this paper.

However, the estimation of the ice drift direction appears to be a non-trivial and complex task of the IM system due to the tendency of the ice to suddenly change its drift speed and angle (tidal loops, arcs, reversals, etc.) [12]. The estimation can be achieved using e.g. drifting buoys data. The task of keeping the vessel aligned with the ice drift could then be devoted to DP operators (DPO) in accordance with those data. Significant research is currently carried out in this domain and several methods are under development, such as underwater [13] and aerial observations [14], for example. Those technologies may also involve sea ice observation using radars [15] and digital image processing as presented in [16]. Nevertheless, such technologies are not yet fully effective and qualified for Arctic operations. Therefore, taking into account the complexity of the ice/ship interaction, it appears interesting to have a function within the DPS for changing the heading of the vessel automatically under the DPO supervision. The primary objective of this function would be to reduce the loads on the hull by maintaining the ship aligned against the ice drift with its bow or stern at all times. Furthermore, as outlined in [17], being head-on against the ice drift in level ice conditions seems to be an unstable equilibrium point for the vessel. Thus, having such functionality seems to be relevant even if the ice drift direction is perfectly determined. Still, only managed ice is studied in this paper, because it is currently considered as the only feasible environment for DP operations in ice.

3. Structure of the automatic heading controller in ice

DPS algorithms are composed of three subsequent stages. First, the estimator receives the measurements from all kind of sensors (DGPS, gyrocompasses, wind sensors, etc.) and computes the filtered state of the vessel. Then, based on these filtered data, the controller computes the required set of forces in order to control the vessel to the DPO's desired set point. Finally, the commanded force vector is allocated to the various actuators on the vessel. The reader may refer to [8] for more details.

In the automatic heading control algorithm the objective is to change the heading of the vessel adequately in order to reduce the magnitude of the global ice loads. The proposed structure of the control laws is presented in Figure 2. The algorithms are based on the work in [8] that has been implemented in the HSVA ice basin [18] and successfully tested in various ice conditions during the DYPIC campaigns. Compared to the original control scheme in [8] the automatic heading controller has been added herein. The inputs of this new block are the calculations of the estimator (which also tracks the external forces in real-time), and the controller (which allows controlling the position of the ship to a desired set point). Based on these inputs, the automatic heading controller computes the best heading for reducing the ice loads. It also leads naturally to a reduction in power consumption and to improvement of the stationkeeping performances, as will be shown later in this paper.

The proposed control scheme is tested in a numerical simulation framework based on the model described in the following section, and then compared to the original control laws with fixed heading set by the operator. The following sections of the paper elaborate on the numerical simulation setups, results and analyses.

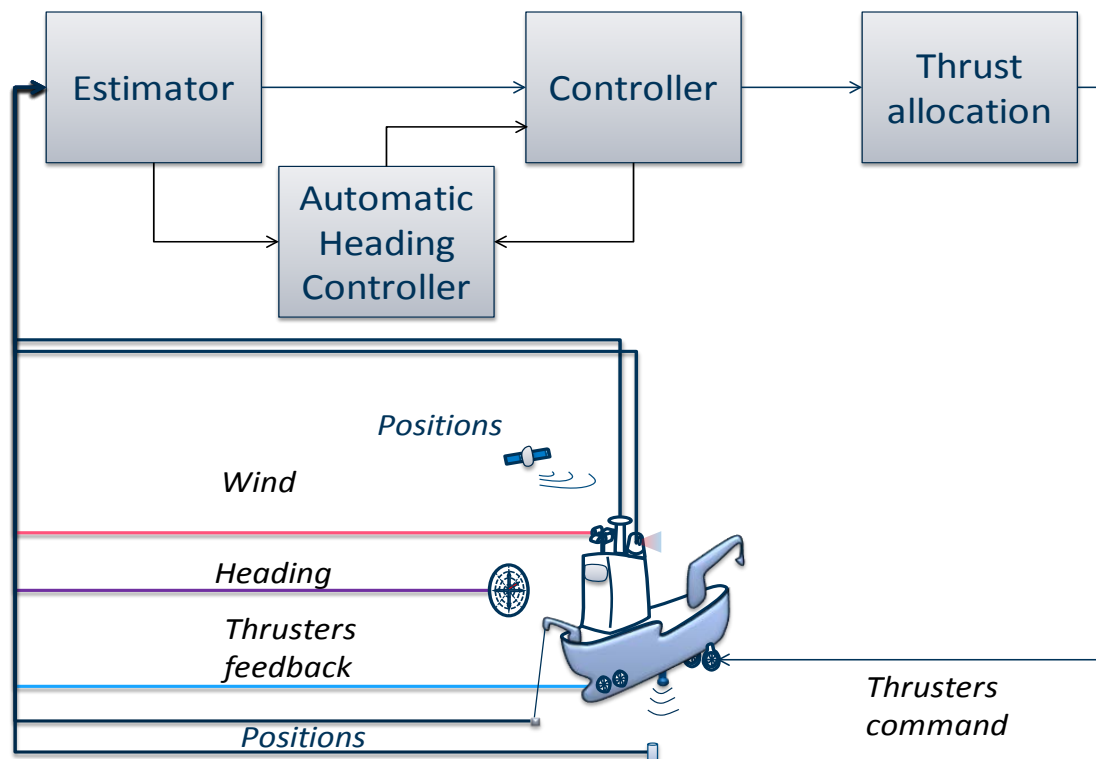


Figure 2. Proposed architecture of the DP system with automatic heading control.

4. Numerical simulation model

The numerical model is a time-stepping simulator based on non-smooth rigid multi-body dynamics with contacts and friction. The free body motion of the objects is described by the Newton-Euler equations, while the contact dynamics is formulated as a linear complementarity problem at the velocity-impulse level with Signorini non-penetration and Coulomb frictional constraints. The complementarity problem is iteratively solved at every timestep using the projected Gauss-Seidel method, while the time integration is performed using the semi-implicit Euler method with timestep equal to 0.03 s. Lastly, possible interpenetrations are corrected using the position projection method. The numerical implementation of the model is based on the NVIDIA PhysX engine [19], and more details on the underlying theory and the methods are given in [20]. PhysX performs collision detection, calculates the contact forces and time-integrates the equations of motions for all objects in the simulation (i.e. the vessel and the ice floes).

The current numerical model was inspired by previous ice-structure interaction models based on PhysX [21, 22] and several other examples of using the physics engine technology for simulating ice-structure interaction, an overview of which can be found in [23]. The present simulator is composed of four interconnected components: the vessel, the ice, the ice tank, and the water volume, and each of the components and their interactions are discussed in the following.

Since drilling is considered as one of the most important exploration activities [1], the vessel employed in the simulations is a conceptual Arctic drillship described in, e.g. [5, 8, 9, 17, 23]. It is simulated as a rigid body with six degrees of freedom (DOFs) with no deformations of the hull. The geometry of the vessel is

represented by a triangle mesh consisting of 2,956 vertices and 5,908 triangles that constitute the detailed structure of the drillship. Although the shape of the vessel is concave, the geometry had to be decomposed into eleven convex parts in order to use it in PhysX for collision detection. Even so, the full triangle mesh was still used for the calculation of the buoyancy forces. The mass of the vessel is 2535 kg, while its inertia tensor is approximated from the geometry using the method of Tonon [24], which assumes a uniform mass distribution.

The water is simulated as a static plane that produces buoyancy and drag loads on the vessel and the ice floes. The implementation is based on the method of Catto [25] with linear and angular drag coefficients equal to 0.4 and 0.6 respectively. The water density is 1000 kg/m³ and the gravitational acceleration is 9.807 m/s².

The ice floes are simulated as rigid bodies in six DOFs, and have the shapes of rectangular cuboids with certain masses and inertia tensors. The masses are calculated from the volumes of the ice floes and the ice density (=900 kg/m³), while a tabulated formula is used for calculating the inertia tensors (inertia tensor for a cuboid). In addition, optimized collision detection algorithms (“box – convex”) are utilized by PhysX to take advantage of the simple shapes of the ice floes and accelerate the simulation. The initial ice field is generated by the simulator in 2D to obtain a certain ice concentration and floe size distribution. Then every ice floe is extruded into 3D by applying a uniform thickness.

The contacts among the various objects in the simulation are calculated dynamically and treated according to the rheology summarized in Table 1.

Table 1. Contact rheology.

Ship-ice friction coefficient	0.0976/0.11 (dyn/stat)
Ice-ice friction coefficient	0.2/0.3 (dyn/stat)
Ice-basin wall friction coefficient	0.2/0.3 (dyn/stat)
Coefficient of restitution	0.2

5. Numerical simulations

This section benchmarks the automatic heading controller (ADPS) against the fixed heading controller (FDPS) in ice tank simulations as described in the previous section. The tuning and the gains of the common blocks of ADPS and FDPS are identical in the complete set of simulations. Furthermore, the tuning of the automatic heading controller is also kept similar in order to prove the robustness of the proposed concept.

The simulation scenario is the same as in [9] and is shown in Figure 3.

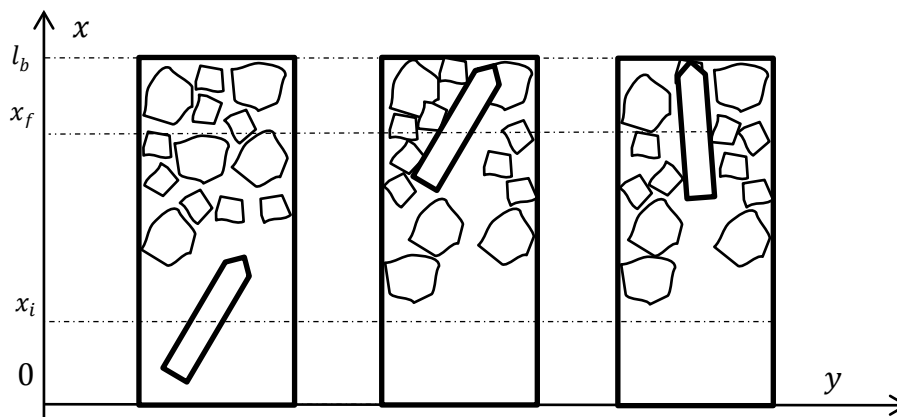


Figure 3. Schematic overview of the simulation scenario. left – start of the simulation; center – end of the simulation with the fixed heading system; right – end of the simulation with the automatic heading strategy.

The simulation starts with the vessel positioned at x_i . Then, the main carriage of the virtual ice tank starts to move in order to reproduce the ice drift with the desired velocity up to the final position x_f . The aim of the FDPS is to reach this final position and keep the heading at its initial value while also keeping the relative position to the main carriage (i.e. simulating a stationkeeping operation). For the ADPS, the objective is not only to follow the main carriage without any positional deviations, but also try to reduce the angle between the ship and the ice drift during the whole simulation. In this setup, the “best” heading to be reached by the ADPS is obviously 0° . In order to cope with the transients due to the carriage acceleration and deceleration, the figures and all numerical analyses in the following sub-sections consider only the samples where the carriage travels at its desired velocity (which is constant and equal to the ice drift velocity). The numerical simulations have been performed in model-scale, but all numerical values (positions, forces, power) are presented in full-scale units in this paper, using the Froude scaling laws with a factor of 30 (according to the DYPIC model tests).

5.1. Simulation scenarios

In order to have a base case for comparing the two strategies, all simulations in this paper are performed in the same model-scale ice field. It has a concentration of $8/10^{\text{th}}$, rectangular ice floes with sizes ranging between 3 and 15 meters at fullscale – i.e. between 0.1 and 0.5 m in the simulation at model scale- and the ice thickness of 1 meter fullscale – i.e. 3.3 cm model scale. The dimensions of the virtual ice tank are 92 m by 10 m model scale and a screenshot of the simulated conditions is shown in Figure 4.

Three different scenarios have been investigated, as shown in Table 2. The ice drift angle is increased between the scenarios, in order to investigate the severity of the increased ice forces on the hull.

	Drift angle ($^\circ$)	Ice drift velocity model scale (m/s)	Ice drift velocity full scale (knots)
Scenario 1	10	0.047	0.5
Scenario 2	30	0.094	1
Scenario 3	50	0.094	1

Table 2. Scenarios summary.

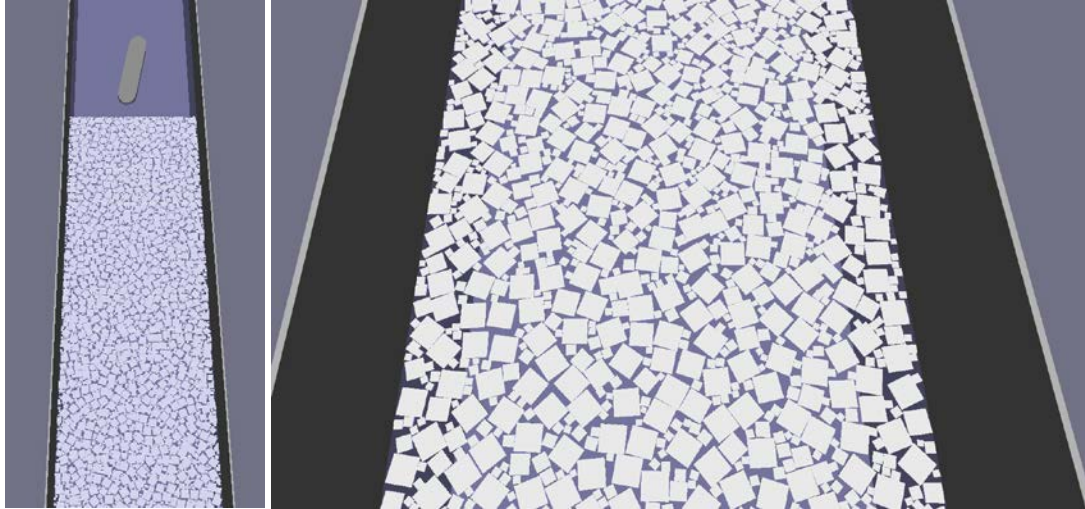


Figure 4. Simulated ice conditions.

5.2. Results

5.2.1. Scenario 1

The positions of the vessel in both simulations (ADPS and FDPS) are shown in **Figure 5** relative to the towing carriage. **Figure 6** shows the evolution of the heading during the simulations. The estimated forces acting on the hull, as defined and computed by the DP estimator, are shown in **Figure 8**. Finally, **Figure 7** represents the power consumption of the thrusters. Some statistical properties of the presented signals are given in Table 3.

As expected, the heading is close to 0° at the end of the simulation with the ADPS control. The stationkeeping in both cases is accurate even though the standard deviation of the position signal is slightly higher with the FDPS. The ice forces, especially the transversal force, are reduced with ADPS. Finally, a reduction of 10% of the accumulated power consumption during the entire simulation can be noted with ADPS control.

5.2.2. Scenario 2

The positions of the vessel in both simulations (ADPS and FDPS) are shown in **Figure 9** relative to the towing carriage. **Figure 10** shows the evolution of the heading during the simulations. The estimated ice forces are represented in **Figure 12**. Finally, **Figure 11** represents the power consumption of the thrusters. Table 4 summarizes the considered statistical properties of the presented signals.

In both cases the DPS have maintained the position of the vessel satisfactorily and the stationkeeping performances have the same order of magnitude. However, the ADPS is performing a little better than FDPS, as in scenario 1. As expected, the heading at the end of the simulation with the ADPS is close to 0° . However, the analysis of the force signals shows that the ADPS is reducing the ice loads significantly, both in terms of magnitudes and in terms of variations. The reduction of the transversal forces is especially notable. It leads inexorably to substantial reductions in power consumption, accounting to ~80%.

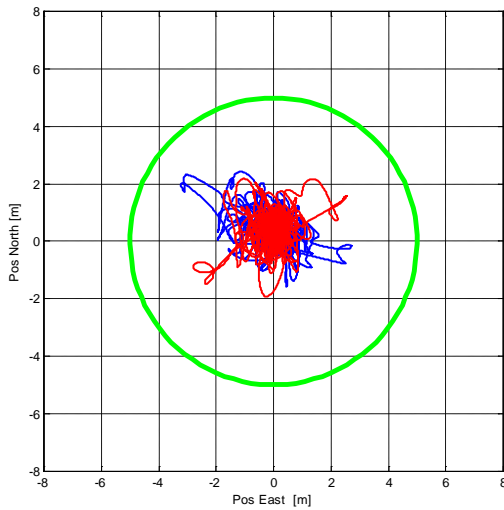


Figure 5. Position of the vessel in Scenario 1: blue – FDPS; red – ADPS; green – 5 meters circle.

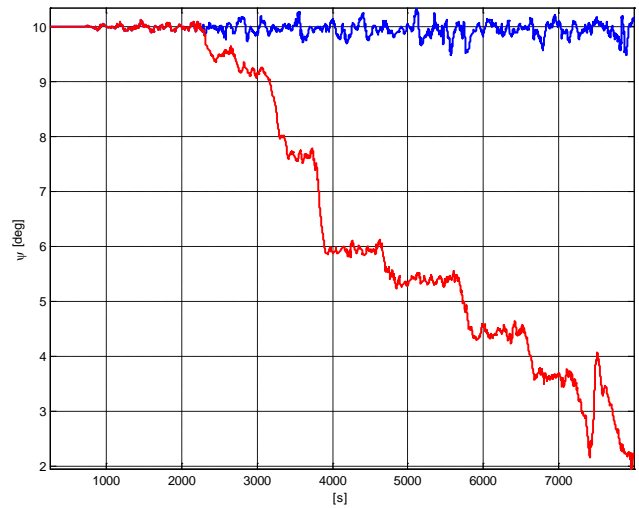


Figure 6. Heading of the vessel in Scenario 1: blue – FDPS ; red – ADPS.

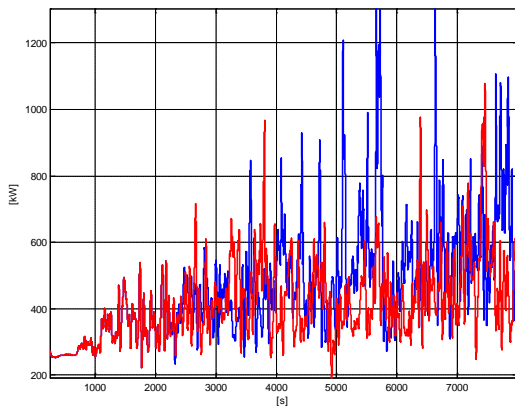


Figure 7. Power consumption in Scenario 1: blue – FDPS; red – ADPS.

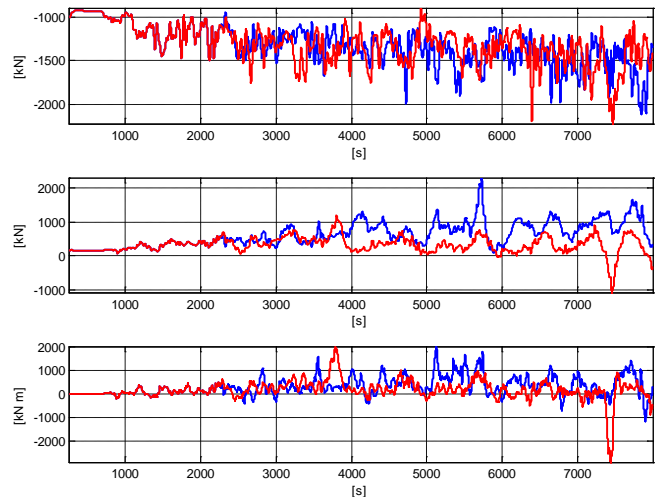


Figure 8. Forces on the hull in Scenario 1: top – surge; center – sway; bottom – yaw torque; blue – FDPS; red – ADPS.

	Standard deviation		Median value	
	FDPS	ADPS	FDPS	ADPS
Pos North [m]	0.55	0.59	0.39	0.41
Pos East [m]	0.64	0.57	0.02	0.02
Longitudinal Force [kN]	216.3	487.4	-1307.0	-1282.5
Transversal Force [kN]	359.4	233.5	567.4	323.2
Yaw Torque [kNm]	393.2	409.9	248.3	134.3
Power Consumption [kW]	1647	1228	4371	3913

Table 3. Signal analysis of Scenario 1.

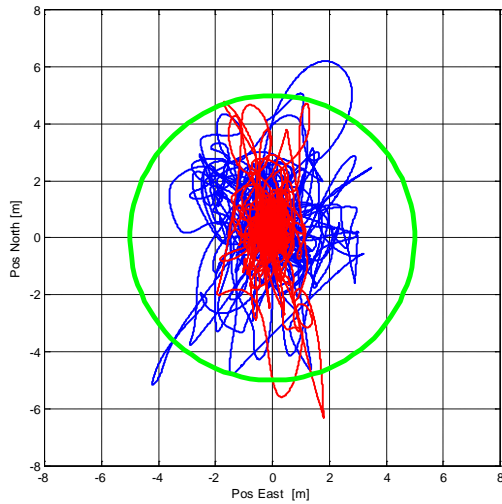


Figure 9. Position of the vessel in Scenario 2: blue – FDPS; red – ADPS; green – 5 meters circle.

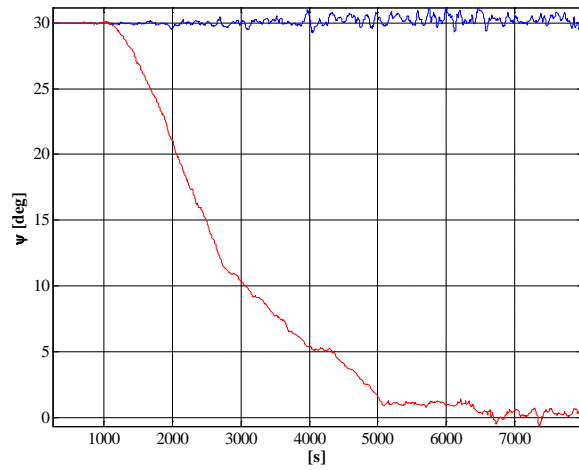


Figure 10. Heading of the vessel in Scenario 2: blue – FDPS; red – ADPS.

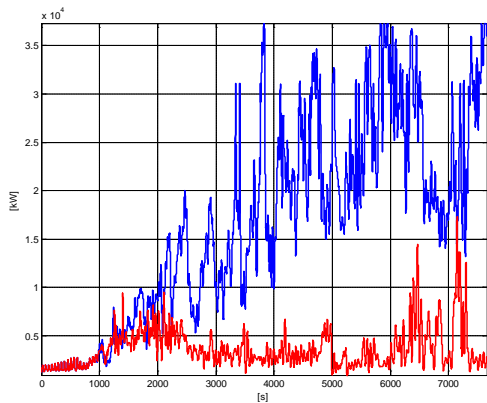


Figure 11. Power consumption in Scenario 2: blue – FDPS; red – ADPS.

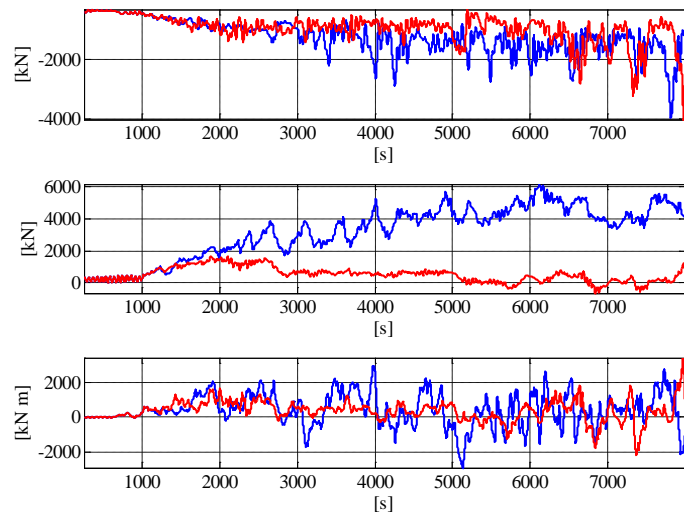


Figure 12. Forces on the hull in Scenario 2: top – surge; center – sway; bottom – yaw torque; blue – FDPS ; red – ADPS.

	Standard deviation		Median value	
	FDPS	ADPS	FDPS	ADPS
Pos North [m]	1.42	1.10	0.24	0.24
Pos East [m]	1.20	0.50	-0.03	0.07
Longitudinal Force [kN]	557.4	441.1	-1168.4	-841.8
Transversal Force [kN]	1631.5	457.4	3625.8	491.1
Yaw Torque [kNm]	918.2	551.9	379.6	390.4
Power Consumption [kW]	10366	2053	16599	3139

Table 4. Signal analysis of Scenario 2.

5.2.3. Scenario 3

In this scenario, the vessel with FDPS loses its position and heading. The drift-off is obviously caused by the magnitude of the ice forces, which are too strong to cope with. It can be seen in **Figure 15** that all the power is used without success for maintaining the position. Controlled by the ADPS, the vessel is able to keep its position quite accurately. The maximal positional error is less than 5 meters and again at the end of the simulation the vessel's heading is close to 0° which demonstrates well the concept of the Automatic Heading controller. The efforts on the hull and the consumed power are considerably reduced once the vessel starts to be well aligned with the ice drift. This scenario demonstrates clearly the enhanced stationkeeping abilities of the ADPS compared to the FDPS.

6. Discussion

The presented simulations demonstrate that the automatic heading controller is achieving its goal. At the end of the simulations, the vessel controlled by the ADPS always reaches a heading close to 0° . This leads to better stationkeeping performance, compared to the system with fixed heading. Table 5 presents the reduction ratios for the median values of the transversal load, as well as for the consumed power. It can be seen that the reduction ratio of the ADPS is higher in tough ice conditions, i.e. scenarios 2 and 3.

	Scenario 1			Scenario 2			Scenario 3		
	FDPS	ADPS	Ratio	FDPS	ADPS	Ratio	FDPS	ADPS	Ratio
Median of the transversal force [kN]	567.4	323.2	43%	3625.8	491.1	86%	6478.7	889.6	86%
Median of the power consumption [kW]	4371	3913	10%	16599	3139	81%	34200	8012	77%

Table 5. Summary of the simulation scenarios.

Such ice load reduction may be of interest for DP operations in the Arctic in several ways. First of all, this system enhances the stationkeeping capabilities of the ship. Further, it reduces dramatically the power consumption and therefore decreases the operational expenditures of the vessel. It also increases consequently the self-sustainability of an operation, which is especially relevant for Arctic offshore scenarios. Moreover, aligning the ship with the ice drift may help in avoiding some threats, such as reducing the exposure of the ship side to multiyear floes and ridge fragments. Finally, it can extend the ship life and also increase the safety of the operations. Nevertheless, some operational issues may occur with such a system, e.g. in certain drilling operations the heading of the vessel may not be completely free.

It is important to emphasize that the presented system has only been tested in numerical simulations. The numerical model has certain limitations and its validity domain has not been fully qualified. For example, one of the uttermost limitations of the numerical model is that it does not simulate the ice fracture and fragmentation. Therefore, the physical validity of such numerical model can be questioned. The interested reader is referred to e.g. [5] for a broader discussion on the modeling approach and its limitations. Finally, the proposed control system has only been tested with a constant ice drift direction, while in the Arctic there may be tidal loops and arcs.

It can be noted that the presented concept may be potentially extended to thruster-assisted mooring operations. Specifically, it can be e.g. an improvement of the system set up by Zhou et al. in [26] for the control of a moored icebreaking tanker. However, the effect of the mooring on the design and tuning of the automatic heading controller must be investigated in detail first.

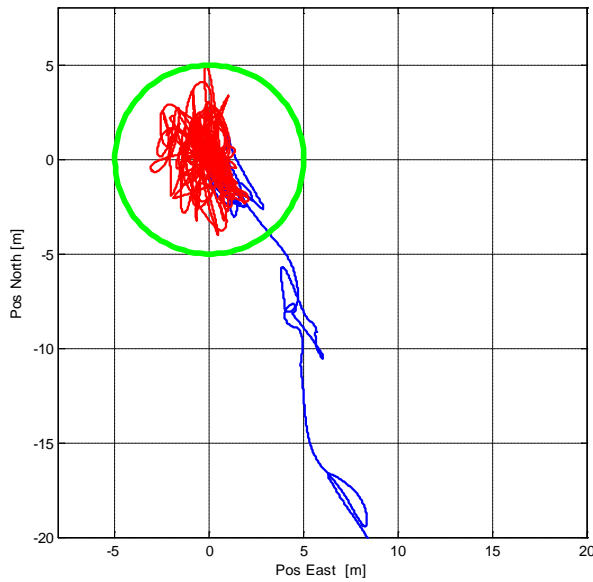


Figure 13. Position of the vessel in Scenario 3: blue – FDPS; red – ADPS; green – 5 meters circle.

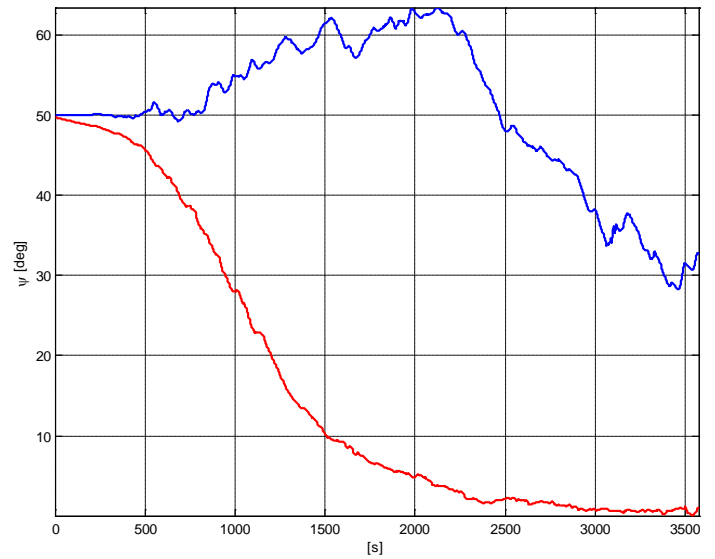


Figure 14. Heading of the vessel in Scenario 3: blue – FDPS; red – ADPS.

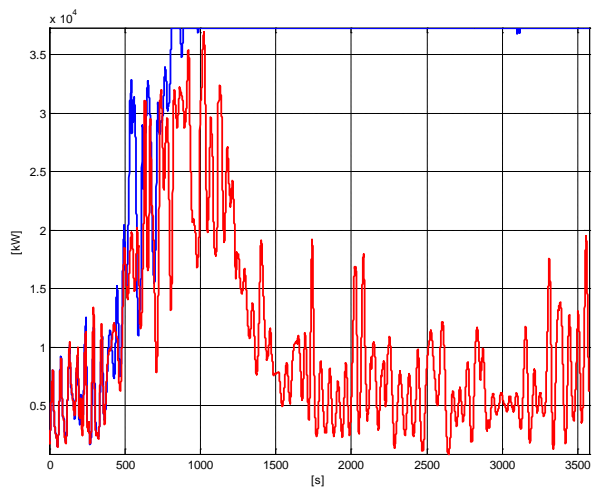


Figure 15. Power consumption in Scenario 3: blue – FDPS ; red – ADPS.

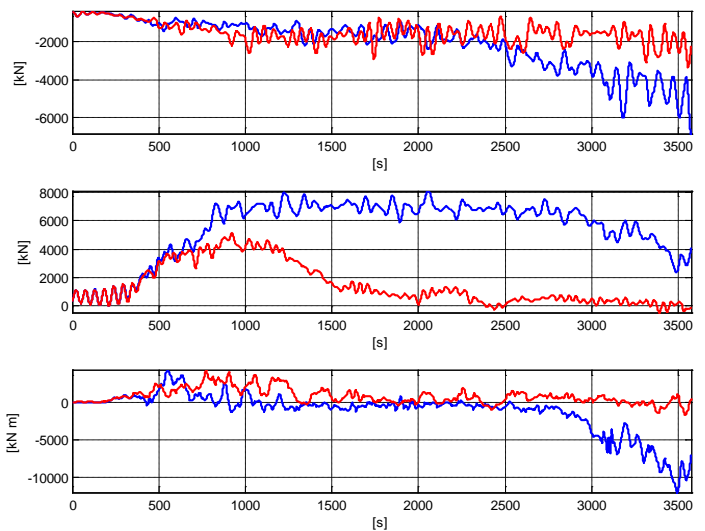


Figure 16 : Forces on the hull in Scenario 3: top – surge; center – sway; bottom – yaw torque; blue – FDPS ; red – ADPS.

	Standard deviation		Median value	
	FDPS	ADPS	FDPS	ADPS
Pos North [m]	--	1	--	0.41
Pos East [m]	--	1.5	---	-0.19
Longitudinal Force [kN]	1281.8	551.0	-1631.8	-1541.2
Transversal Force [kN]	2101.6	1463.2	6478.7	889.6
Yaw Torque [kNm]	2787.8	1028.7	-441.2	631.0
Power Consumption [kW]	10924	7956	34200	8012

Table 6. Signal analysis of Scenario 3.

7. Conclusions

This paper reviewed the challenges of dynamic positioning operations in ice and motivated the need for an automatic heading function that could maintain the ship aligned with the ice drift direction and minimize the ice loads. A structure of a controller addressing this issue has then been proposed and tested in ice basin numerical simulations with a high-fidelity numerical tool developed by NTNU. The simulation results show good performances of the concept in a set of selected scenarios, namely the enhanced stationkeeping capacities compared to the system with constant heading. Furthermore, significant reductions of the ice loads are observed with the new strategy, resulting in considerable power consumption cuttings. The automatic control of the heading may therefore be considered as a new asset for the development of dynamic positioning systems for ice operations.

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