THE EFFECT OF MEMORY IN PASSIVE NONLINEAR OBSERVER DESIGN FOR A DP SYSTEM

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ABSTRACT

The behavior of marine structures is investigated in regular waves using various models in frequency and time domains. In normal practice, the differential form of motion equations are used with constant hydrodynamic coefficients. Those coefficients which are added masses and damping coefficients are frequency dependent due to the memory effect. The memory effects are usually represented by a convolution integral. This term makes the motion equations in integro-differential forms. This makes the equation complex and needs special attention. However, it is necessary to take into account the memory effect in ship motion control and specially in dynamic positioning system to enhance the accuracy and obtained a more precise controller. The motion equations are solved for a DP system with memory effect and compare with the conventional model. The computation shows that memory effect is important as the wave frequency increases. A nonlinear observer has been design to solve the wave filtering and state estimation problems with the memory effect. The observer has been proven to be passive and global exponential stability.

Keywords: memory effect, Time-domain model, nonlinear observer.

INTRODUCTION

Dynamic positioning (DP) is a controlled system which automatically maintain vessel's position and heading only by means of propellers and active thrusters. The design of a proper dynamic positioning system for vessel is dependent on the accuracy of the adopted model. Conventional control systems for dynamic positioning are based on mathematical models with constant added mass and damping coefficients [1, 2]. The concept of frequency-dependent added mass and damping coefficient is well established in the formulation of the ship motion equations by Ogilvie [3]. Cummins shows this dependency with a convolution term in the radiation potential [4]. This term is known as memory effect. Kernel of this term is the impulse response of the system. It is necessary to obtain added masses and damping coefficients for various frequencies to compute the impulse response function. The added masses and damping coefficients can be computed by using 2D or 3D hydrodynamics codes based on the potential theory.

The formulations of the equation of motion based on the use of convolution terms are not in agreement with the model formulations used in controller design [5, 6, 7]. Different methods have been proposed in the literatures as approximate replacement of the convolutions [5-7].

Position measurements of ship are noisy. Moreover, some of its states are not available. Consequently filtering and state estimation are important in DP systems design. Early observer design for DP systems focused on applying linear observer theory such as Kalman filters. Linearization of the nonlinear model cause that observer be valid locally. It is drawback of Kalman filter. A nonlinear passive observer has been proposed by [8]. The observer considered slowly-varying forces such as 2nd-order wave drift, wind, ocean currents. Moreover, reconstruction of the low frequency (LF) motion took into account.

The environmental disturbances acting on a ship induce at two separate motions. The first-order wave forces induce high-frequency motions while slowly-varying forces induce low-frequency motions. Dynamic Positioning System must cancel out constant and the slowly varying disturbances to keep the vessel mean position as close as possible to the desired position. Early DP systems were designed using PID controllers. However, these controllers exhibit phase lag and performance loss. These drawbacks can be removed by applying a complete nonlinear Dynamic Positioning System with a nonlinear observer and a backstepping controller according to [9].

The memory effect is taken into account to design the observer for DP Systems. The memory effect is represented by a convolution integral and accordingly, an integro-differential formulation is applied to model the vessel’s dynamics. The method of Kristiansen et al. [5] is applied to approximate the computationally expensive convolution terms with a state space representation while the added masses
and damping coefficients are obtained by using the conformal mapping. The design observer is an extension to the passive nonlinear observer of Fossen et al. [8] with considering memory effect in dynamic model. It is assumed that the position measurements of the vessel are the only available data. The performance of the nonlinear observer is demonstrated by computer simulations. It has been shown that the accuracy of nonlinear observer is enhanced by taking into account the memory effect.

Memory effect

When the body is forced to oscillate, waves will propagate outward from the body. This will affect the fluid pressure and change the fluid momentum hence the body force for all subsequent times [10]. This is initiate memory effect. From the other point of view, the hydrodynamics coefficients are frequency dependent that is shown in Fig. 1. By considering this dependency the ship motion equation in body-fixed frame may be given in the following form [3]

\[
\begin{align*}
\left[ M_{RB} + A(\omega) \right] J^T (\eta) \ddot{\eta}(t) + B(\omega) J^T (\eta) \dot{\eta}(t) + G J^T (\eta) \eta(t) &= f_{ext} \\
\end{align*}
\]

(1)

This form is valid if all external forces be sinusoidal. Moreover, this model is not suitable for observer and controller design because its coefficients are frequency dependent [6, 7].

The more suitable model is the Cummins formulation [4] in time domain.

\[
\begin{align*}
\left[ M_{RB} + A \right] J^T (\eta) \dot{\eta}(t) + \int_0^t K(t - \tau) J^T (\eta) \dot{\eta}(\tau) d\tau \\
+ G J^T (\eta) \eta(t) &= f_{ext} \\
\end{align*}
\]

(2)

radiation forces modeled as the following form

\[
\begin{align*}
f_{rad} = -A_x \ddot{\eta} - \int_0^t K(t - \tau) J^T (\eta) \dot{\eta}(\tau) d\tau \\
\end{align*}
\]

(3)

The first term related to pressure force due the acceleration of the structure, and \( A_x \) is a constant positive-definite added mass matrix [11]. The second term that is the convolution term is known as a
fluid-memory model. The kernel of the convolution term, $K(t)$, is the impulse response to the relative velocity between ship and wave.

If assume that the vessel motion is sinusoidal $x_k(t) = x_k \cos(\omega t)$ $x_k(t) = 0$ for $t<0$, then the convolution integral can be expressed as follows [3].

$$K_{jk}(t) = -\frac{2}{\pi} \int_0^\infty [A_{jk}(\omega) - A_{jk}(\infty)] \sin(\omega t) d\omega$$

(4)

$$K_{jk}(t) = \frac{2}{\pi} \int_0^\infty [B_{jk}(\omega) - B_{jk}(\infty)] \cos(\omega t) d\omega$$

(5)

Hydrodynamic coefficients calculation

The added masses and damping coefficients are known as hydrodynamic coefficients. These hydrodynamic coefficients can be obtained experimentally by using planar motion mechanism (PMM) test or theoretically and applying potential theory. The strip theory is the most common method in ship hydrodynamics to obtain the hydrodynamic coefficients. The strip theory is based on the assumptions that the ship is a slender and the wave length is of order of the ship beam. The flow field can be approximated at any section of the ship by assumed two-dimensional flow in a cylindrical strip of length $dx$. The total effects of all individual strips are integrated along the ship length.

We apply the strip theory to calculate the added masses and damping coefficients. These hydrodynamic coefficients for 2D sections are obtained using the conformal mapping with Lewis forms. We also applied the FDI toolbox of Thor I. Fossen and Tristan Perez [11] to compute the added mass for infinite quantities.

The impulse response function is obtained by using equation (9). We calculate frequency dependent added mass for frequencies up to 3 rad/s. For higher frequencies, an extrapolation of the data by a polynomial curve fitting has been used.

Convolution replacement

In Cummins equation frequency dependency of hydrodynamics coefficients have been replaced by a convolution term even so this term is not convenient to analysis and design of motion control systems. Different methods have been proposed in the literature as approximate alternative representations of the convolutions. State-space models are well suited for the methods of analysis used in automatic control. To replace the convolution terms with a state-space representation, Kristiansen et al method has been used [5]. Convolution term define as the following form

$$\mu_j(t, \tau) = \sum_{k=1}^{6} \int_0^\infty K_{jk}(t - \tau) v_k(\tau) d\tau$$

(6)

With the vector $\mu = [\mu_1, \ldots, \mu_6]^T$. We know that the convolution is a linear term. Define a state space model as the following form

$$\dot{\xi} = A\xi + B\dot{x}$$

$$\mu = C\xi + D\dot{x}$$

(7)
Where $\xi$, $\dot{x}$, $\mu$ are state vector, input vector and output vector, respectively. Replace output of this model, $\mu$, into the equation [2]

$$[M + A]\ddot{x}(t) + \mu(v, t) = f_{ext} \tag{8}$$

3 DOF Model

**Kinematic equation of motion**

To explain ship motion, it is convenient to define two coordinate. The earth-fixed coordinate which vessel’s position and orientation are measured relative to a defined origin.

$$\eta = [x, y, z, \phi, \theta, \psi]^T \tag{9}$$

And the body fixed frame that is fixed to the vessel. Velocities are measured relative to this coordinate

$$\nu = [u, v, w, p, q, r]^T \tag{10}$$

The kinematic equation of motion is related to transformation between the body-fixed and the earth-fixed velocity vectors [1].

$$\eta = J(\eta)\nu \tag{11}$$

Where $J(\eta)$ is transformation matrix which dependent on Euler angles.

Surface vessels vertical motions are zero mean oscillatory and their amplitude are limited. Therefore, it is common to study ship motion control in horizontal plane include sway, yaw and surge modes. In 3DOF, position vector is $\eta = [x, y, \psi]^T$ and the velocities in body-fixed reference frame be represented by the vector $\nu = [u, v, r]^T$. Transformation matrix in horizontal plane only dependent on yaw angle

$$J(\eta) = J(\psi) = \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{12}$$

**Kinetic model**

The model in the horizontal plane is found by separating the surge, sway and yaw elements and simultaneously neglecting heave, roll and pitch to zero.

The dynamics model is written as the following form

$$[M + A]\dot{\nu}(t) + \mu(v, t) = f_{ext} \tag{13}$$
Observer design

Position measurements of ship are noisy. Moreover, some of its states are not available. Filtering and state estimation are important aspects in DP systems design. In these systems, the main objective of the observer is to provide low-frequency estimates of the vessel’s positions, heading, and velocities. The observer will also be needed to predict the motion of the vessel in situations where position or heading measurements become unavailable [8].

The conventional solution for the wave filtering and state estimation problem in Dynamic Positioning Systems had been using linear methods for example notch filters and Kalman filters. The major disadvantage of linear methods is its local validity [8]. Fossen et al (1999) have proposed a passive nonlinear observer. The observer includes wave filtering properties, bias state estimation, reconstruction of the LF motion components and noise free estimates of the non-measured vessel velocities. The proposed observer has been proven to be passive and globally exponentially stable (GES) [8]. Here, we extended this observer by considering memory effect in ship dynamics. The performance and robustness of the observer are verified by computer simulations.

To design an observer we should define complete model of the system. This model includes kinematic and kinetic equations position and heading measurement equations, 1nd-order wave induced motion model and bias model. Kinematic and kinetic equations were described. Now explain rest models from [8].

Measurement equation

For usual ships, position and heading measurements are measured by differential global positioning system (DGPS) or hydroacoustics positioning reference (HPR) systems, and gyro compasses. The total measurement equation can be written as

\[ y = \eta + \eta_{\omega} + v \]  

Where \( \eta_{\omega} \) is the vessel’s WF motion due to 1st-order wave-induced disturbances and \( v \in \mathbb{R}^3 \) is zero-mean Gussian white measurement noise [8].

Bias Modeling

Bias model can be used to describe slowly-varying environmental forces and moments due to (Fossen 1994) 2nd-order wave drift, ocean currents, wind. A commonly used bias model for marine control applications is the 1st-order Markow process [8]

\[ \dot{b} = -T^{-1}b + \psi_n \]  

1st-Order Wave Induced Model

The 1st-order wave induced model approximation that is include damping term was proposed by Salid and Jenssen (1983). State space representation of this model in 3DOF is

\[ \dot{\xi} = \Omega \xi + \Sigma \omega \]
\[ \eta_{\omega} = \Gamma \xi \]
Where $\xi \in \mathbb{R}^3$, $\omega \in \mathbb{R}^3$ and Order of this model is related to desired precise approximation to the actual wave spectrum.

**Observer equations**
Based on equations (22-26) and neglecting the disturbance terms, the nonlinear observer can be formed as

$$
\dot{\xi} = \Omega \xi + K_1 \tilde{y} \\
\dot{\eta} = J(y) \tilde{v} + K_2 \tilde{y} \\
\dot{b} = -T^{-1} b + \frac{1}{\gamma} \Lambda \tilde{y} \\
[M + A] \tilde{v} = -D \tilde{v} + J^T(y) \tilde{b} + \tilde{\mu}(t, \nu) + \frac{1}{\gamma} J^T(y) K \tilde{y} \\
\tilde{y} = \eta + \Gamma \tilde{\xi}
$$

(17) \hspace{1cm} (18) \hspace{1cm} (19) \hspace{1cm} (20) \hspace{1cm} (21)

Where $K, K_1, K_2$ and $\Lambda$ are observer gains with appropriate dimensions and $\gamma$ is tuning parameter [8]. These quantities are computed by using Lyapunov function. The performance of the nonlinear observer is demonstrated by computer simulations.

**Simulations results**
To evaluate the performance of the nonlinear observer computer simulations has been used. Model Rigid body mass, added mass and potential damping matrices are as follows, respectively

$$
M = \begin{bmatrix}
1.2 \cdot 10^8 & 0 & 0 \\
0 & 1.4 \cdot 10^8 & 3 \cdot 10^7 \\
0 & 3 \cdot 10^7 & 3.46 \cdot 10^{11}
\end{bmatrix}
$$

$$
A = \begin{bmatrix}
3 \cdot 10^6 & 0 & 0 \\
0 & 2.1 \cdot 10^7 & 3.5 \cdot 10^7 \\
0 & 3.5 \cdot 10^7 & 5.46 \cdot 10^{10}
\end{bmatrix}
$$

$$
D = \begin{bmatrix}
2 \cdot 10^6 & 0 & 0 \\
0 & 1.210^7 & -7 \cdot 10^6 \\
0 & -7 \cdot 10^6 & 2.23 \cdot 10^{10}
\end{bmatrix}
$$

Observer gains are obtained by means of algebraic Lyapunov matrix equation [8].
\[
K_1 = \begin{bmatrix}
-4.32 & 0 & 0 \\
0 & -4.32 & 0 \\
0 & 0 & -4.32 \\
2.23 & 0 & 0 \\
0 & 2.23 & 0 \\
0 & 0 & 2.23
\end{bmatrix}
\]

\[
K = \begin{bmatrix}
0.1 & 0 & 0 \\
0 & 0.1 & 0 \\
0 & 0 & 0.01
\end{bmatrix}, \Lambda = 0.1K
\]

\[
K_2 = \begin{bmatrix}
1.2 & 0 & 0 \\
0 & 1.2 & 0 \\
0 & 0 & 1.2
\end{bmatrix}
\]

And \( \gamma = 1 \).

The bias time matrix was chosen as:

\[
T = \begin{bmatrix}
1000 & 0 & 0 \\
0 & 1000 & 0 \\
0 & 0 & 1000
\end{bmatrix}
\]

**Thrust Allocation**

The model has been equipped with 3 azimuth thrusters that can be rotated to any azimuth angle. Thruster’s arrangement is shown in fig. 2.

Fig. 2  Schematic of thruster arrangement of a vessel equipped with 3 azimuthing thrusters

The thruster configuration matrix is as follows
The control inputs were chosen as:

\[
\tau = \begin{bmatrix}
7 \cdot 10^5 \sin(0.02t) \\
7 \cdot 10^5 \sin(0.05t) \\
8 \cdot 10^6 \sin(0.04t)
\end{bmatrix}
\]

Then thrust for each thruster is computed from following relation [14].

\[
T_a = T^{-1}(\alpha) \tau
\]  

(22)

The simulation results are shown in Figs 3-8. Figs. 3-5 are shown impulse response functions. And Figs. 6-8 are related to estimated position and velocity.
Fig. 5. Yaw-yaw impulse response function

Fig. 6. Estimated x filtered by observer

Fig. 7. xw(wave induced motion)
CONCLUSIONS
A nonlinear observer has been extended for ship motions in the horizontal plane. The nonlinear observer has included estimation of slowly-varying disturbances, low-frequency position and low-frequency velocity of the ship from noisy measurements. The observer is design by using integro-differential equations of motion that includes memory effect. The inclusion of the memory effect is the main advantage of the observer. The memory effect is represented by a convolution integral in the motion equations. The convolution term is replaced by state space representation for control application. Consideration of memory effect increases accuracy of results especially in rough seas with varying frequency.

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REFERENCES
Integral Action and Wave Filtering for Ships”, Proceedings of the IFAC Conference on control
Application in marine systems (CAMS’98), Fukuoka, Japan, 27–20 October
Frequency-domain Data Enforcing Model Structure and Parameter Constraints”. ARC Centre of
Excellence for Complex Dynamic Systems and Control, pp. 1–28
B.V.
for Ocean Technology (IOT)