Dynamic Positioning Committee
Marine Technology Society

Control Systems II

HIERARCHICAL PRINCIPLE of CONSTRUCTION of SPATIAL DYNAMIC POSITIONING (DP) SYSTEMS of SEA MOBILE OBJECTS (SMO)

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-task of DP SMO in space at performance of saving works;

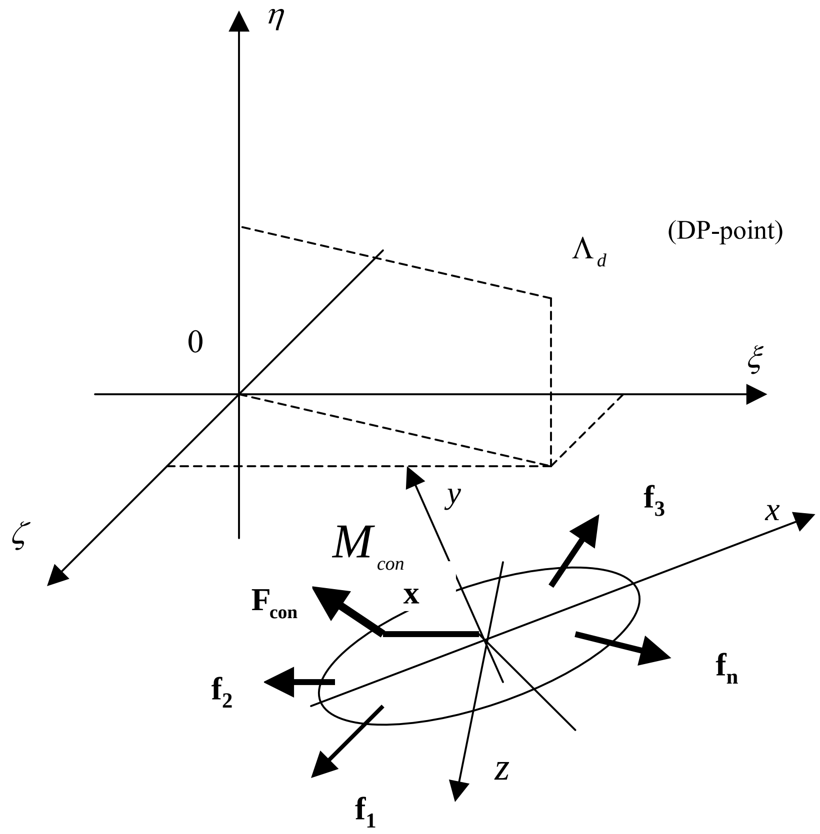
-task of DP of underwater vehicles and robots in space at performance

of different works on shelf and many others similar works.

The statement of task

- ***The basic idea of approach consist in using of two levels control of DP:***
- -at first level (named top level) System of DP define vectors of control force Fcon and moment Mcon ;
- -at second level (named bottom level) System of DP distribute this vectors on thrusters.

For forming Lyapunov function the next deviations are used :



$$s_1 = v - v_d \qquad v_d = 0, \quad s_1 = v$$

$$s_2 = \Lambda - \Lambda_d$$

$$v^T = (v_x, v_y, v_z, \omega_x, \omega_y, \omega_z)$$

$$\Lambda^T = (\xi, \eta, \zeta, \theta, \varphi, \psi)$$

top level of control consist in definition

$$L_{con} = [F_{con}, M_{con}] = ?$$

$$M_{con} = \mathbf{x} \cdot \mathbf{F_{con}}$$

bottom level of control consist in decision of the next ve ctor equations

$$\mathbf{F}_{con}(t) = \sum_{\mathbf{i=1}}^{\mathbf{N}} \mathbf{f}_i(t); \qquad \mathbf{f_i} = ?$$

$$\mathbf{M}_{con}(t) = \sum_{\mathbf{i=1}}^{\mathbf{N}} \mathbf{x}_i \cdot \mathbf{f}_i(t).$$

Control of top level

•***For definition of control laws of top level the next equations of motion SMO are used:***

$$M\dot{v}+C(v)v+D(v)v+g(\Lambda)=L_{con}+L_{w},\qquad v^{T}=(v_x,v_y,v_z,\omega_x,\omega_y,\omega_z)\qquad \Lambda=(\xi,\eta,\zeta,\theta,\varphi,\psi),$$

$$\dot{\Lambda}=H(\Lambda)v,$$

Matrix of Corioliss and centripetal forces

$$C(v)=\begin{bmatrix}A\\B\end{bmatrix}\qquad A=\begin{bmatrix}0 & -m_y\omega_z & m_z\omega_y & 0 & 0 & 0\\ m_x\omega_z & 0 & -m_z\omega_x & 0 & 0 & 0\\ -m_x\omega_y & m_y\omega_x & 0 & 0 & 0 & 0\end{bmatrix}\qquad B=\begin{bmatrix}0 & 0 & 0 & 0 & 0 & 0\\ (m_x-m_z)v_z & 0 & 0 & (J_x-J_z)\omega_z & 0 & 0\\ 0 & (m_y-m_x)v_x & 0 & 0 & (J_y-J_x)\omega_x & 0\end{bmatrix}$$

$D(v)$ – Matrix of hydrodynamic forces

Vector of weight and displacement forces

$$g(\Lambda)^{T}=[(Q-G)\sin\psi,(Q-G)\cos\psi\cos\theta,-(Q-G)\cos\psi\sin\theta,-Q(z_Q\cos\psi\cos\theta+y_Q\cos\psi\sin\theta),$$
$$Q(x_Q\cos\psi\cos\theta-y_Q\sin\psi),Q(x_Q\cos\psi\sin\theta+z_Q\sin\psi)],$$

Matrix of cinematic connections

$$H(\Lambda)=\begin{bmatrix}R & 0\\ 0 & S\end{bmatrix},\quad R=\begin{bmatrix}\cos\varphi\cos\psi & \sin\theta\sin\varphi-\cos\theta\cos\varphi\sin\psi & \cos\theta\sin\varphi+\sin\theta\cos\varphi\sin\psi\\ \sin\psi & \cos\theta\cos\psi & -\sin\theta\cos\psi\\ -\sin\varphi\cos\psi & \sin\theta\cos\varphi+\cos\theta\sin\varphi\sin\psi & \cos\theta\cos\varphi-\sin\theta\sin\varphi\sin\psi\end{bmatrix}\quad S=\begin{bmatrix}1 & -\cos\theta tg\psi & \sin\theta tg\psi\\ 0 & \dfrac{\cos\theta}{\cos\psi} & -\dfrac{\sin\theta}{\cos\psi}\\ 0 & \sin\theta & \cos\theta\end{bmatrix}$$

Control of top level1

- ***Method of synthesis***

For designing of control law (first stage) next Lyapunov function candidate are used

$V = \frac{1}{2}\{s_1^T M s_1 + s_2^T K_2 s_2\}$ control system stable, *if* $\dot{V} < 0$, *whence follows*

$L_{con} = C(v)v + D(v)v + g(\Lambda) - K_1 v - H^T(y) K_2 (\Lambda - \Lambda_d)$ *and* $K_1 > 0, K_2 > 0.$

Matrix K_1, K_2 determine time of transfer processes t_{tp} for little $y(\theta, \varphi, \psi)$

For designing the observer (second stage) next equations are used

$$M\dot{\hat{v}} = -K_1 \hat{v} - H^T(y) K_2 (\hat{\Lambda} - \Lambda_d) + \bar{K}_2 (\Lambda - \hat{\Lambda}),$$

$$\dot{\hat{\Lambda}} = H(y)\hat{v} + \bar{K}_1 (\Lambda - \hat{\Lambda}), \quad y = (\theta, \varphi, \psi).$$

Lyapunov function candidate look as

$$W = \frac{1}{2}(\tilde{v}^T M \tilde{v} + \tilde{\Lambda}^T Q_2 \tilde{\Lambda}), \quad \text{where } \tilde{v} = v - \hat{v}, \tilde{\Lambda} = \Lambda - \hat{\Lambda}.$$

observer stable, *if* $\dot{W} < 0$, *whence follows*

$$\bar{K}_2 = H^T(y) Q_2^T, \bar{K}_1 > 0, D - L > 0,$$

$$L - Lipschitz\ const. \quad for\ C(v).$$

Control of top level2

- ***The final stage of synthesis consist in estimation of stability of controlled motion SMO with observer***

For this purpose consider the equations of control motion of SMO in estimations (assume, that errors of estimation for stable observer too little)

$$M\dot{\hat{v}} = -K_1\hat{v} - H^T(y)K_2(\hat{\Lambda} - \Lambda_d) + \overline{K}_2\widetilde{\Lambda},$$

$$\dot{\hat{\Lambda}} = H(y)\hat{v} + \overline{K}_1\widetilde{\Lambda}.$$

Equations for estimation errors of observer

$$M\dot{\widetilde{v}} = -(C(v)v - C(\hat{v})\hat{v}) - D\widetilde{v} - \overline{K}_2\widetilde{\Lambda},$$

$$\dot{\widetilde{\Lambda}} = H(y)\widetilde{v} - \overline{K}_1\widetilde{\Lambda}.$$

Lyapunov function candidate

$$S = \frac{1}{2}(\hat{v}^T M\hat{v} + \hat{\Lambda}^T Q_1\hat{\Lambda} + \widetilde{v}^T M\widetilde{v} + \widetilde{\Lambda}^T Q_2\widetilde{\Lambda})$$

$$Q_1 = diag\{q_{1i}\} > 0, Q_2 = diag\{q_{2i}\} > 0$$

Control motion stable *if* $\dot{S} < 0$, *whence follows*

$K_2 = Q_1^T, \overline{K}_2 = H^T(y)Q_2^T, K_1 > 0, D - L > 0,$ these conditions coincide with previous and

$\overline{K}_1 = diag\{\overline{k}_{1i} = \dfrac{3q_{2i}}{q_{1i}t_{tp}}\},$ that follows from $\dot{\hat{\Lambda}}^T Q_2^T + \Lambda^T Q_1\overline{K}_1 = 0, \Lambda_i = Ce^{-\frac{q_{1i}\overline{k}_{1i}}{q_{2i}}t},$

additional conditions of concording time of transfer processes of SMO control motion and estimation.

Control of bottom level

- ***Consist in decision of task of distribution*** $L_{con}(F_{con}, M_{con})$ ***on thrusters,***

  that equals to decision next system of vector equations (six scalar equations)

$$\mathbf{F}_{con}(t) = \sum_{\mathbf{i=1}}^{\mathbf{N}} \mathbf{f}_i(t);$$

$$\mathbf{M}_{con}(t) = \sum_{\mathbf{i=1}}^{\mathbf{N}} \mathbf{x}_i \cdot \mathbf{f}_i(t).$$

$$\mathbf{f_i} = ? \quad (a)$$

This system may be have:
- no decision (system not controllability);
- solo decision;
- infinitive quantity of decisions.

$$\mathbf{x_i} = \begin{bmatrix} 0 & -z_i & y_i \\ z_i & 0 & -x_i \\ -y_i & x_i & 0 \end{bmatrix} \quad -skew-symmetric\ matrix \quad (special)$$

$$Decisions\,(a)\ exist,\ if\ x_i\ and\ \mathbf{f_i}\ satisfy\ same\ conditions \quad (criterion\ of\ controllability)$$

In case of infinitive quantity of decisions $\mathbf{f_i}$, additional condition are used

$$\min_{f_i} J = \min_{f_i} \frac{1}{2} \sum_{i=1}^{N} \lambda_i(t) \mathbf{f_i}^{\mathbf{T}}(t) \mathbf{f_i}(t) \qquad (criterion\ of\ \min\ power)$$

Such approach allows to connect all thrusters in united (integrated) control system of DP

Control of bottom level1

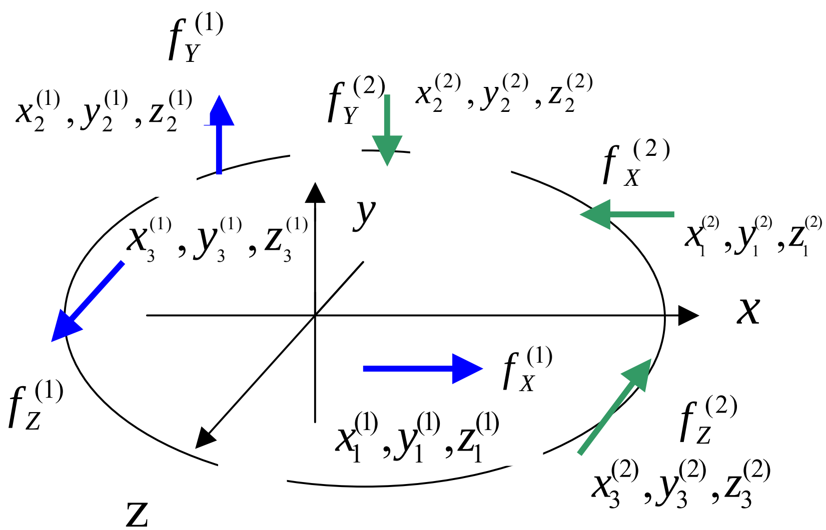
• ***Existing of decisions (solo or infinitive) are determined by criterion of controllability:***

SMO is controlled, if the coordinates of an arrangement of any six thrusters (two groups with three thrusters) with tractions directed along axes (see figure)satisfy to conditions:

$$\left(x_3^{(1)}-x_3^{(2)}\right)\cdot\left(y_1^{(1)}-y_1^{(2)}\right)\cdot\left(z_2^{(1)}-z_2^{(2)}\right)-\left(x_2^{(1)}-x_2^{(2)}\right)\cdot\left(y_3^{(1)}-y_3^{(2)}\right)\cdot\left(z_1^{(1)}-z_1^{(2)}\right)\neq 0$$

and $x_3^{(1)}y_1^{(1)}z_2^{(1)}-x_2^{(1)}y_3^{(1)}z_1^{(1)}\neq 0$

or $x_3^{(2)}y_1^{(2)}z_2^{(2)}-x_2^{(2)}y_3^{(2)}z_1^{(2)}\neq 0$

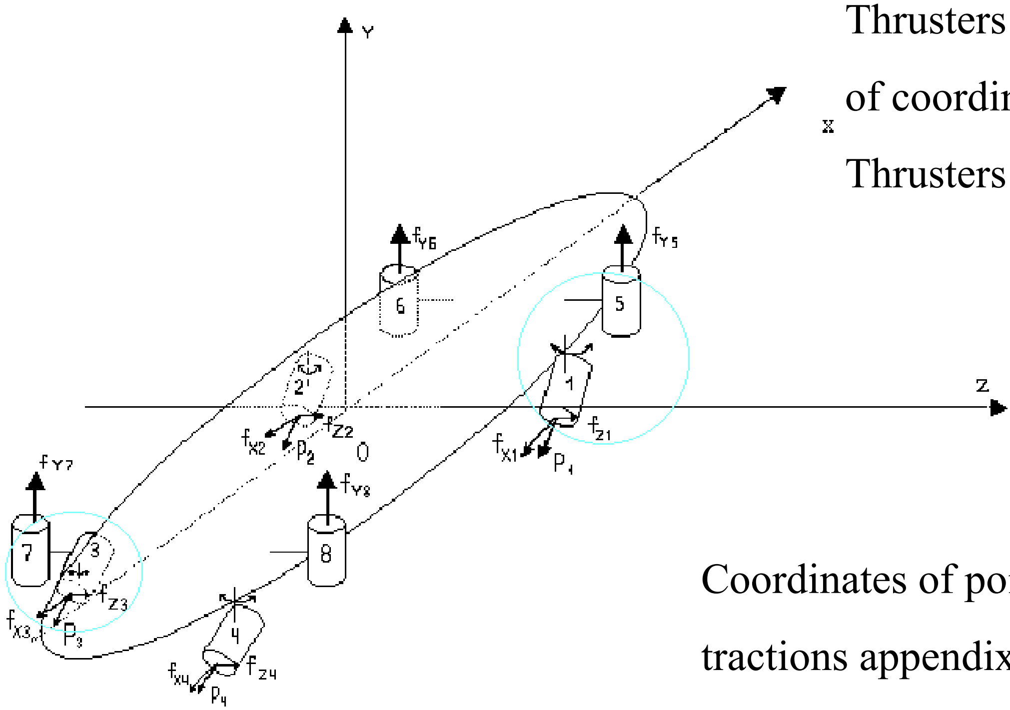


This conditions were concluded after substitution of coordinates of thruster arrangement in equations of task of distribution.

Putting of additional thrusters leads to infinitive quantity of decisions.

Example

- ***Consider the next scheme of an arrangement of thrusters***



Thrusters 1-4 act in horizontal plane in b.f. system of coordinates and can to rotate;

Thrusters 5-8 act along vertical axis.

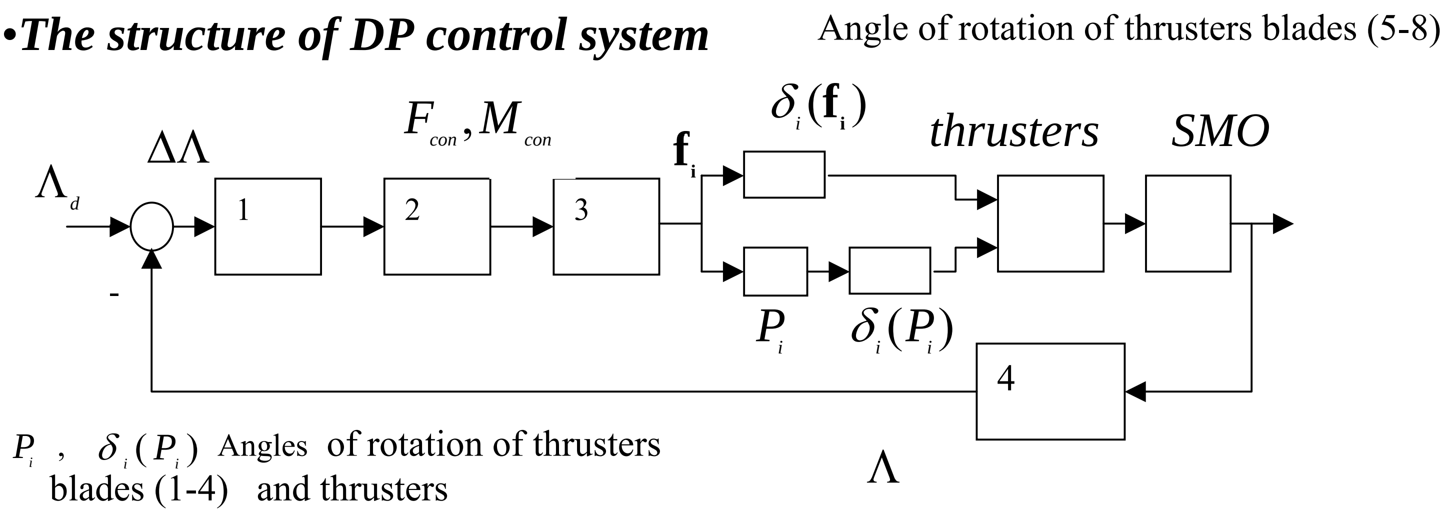
Coordinates of points of the thruster tractions appendix have plane symmetry:

$$1(x_1, y_1, z_1);\ 2(x_1, y_1, -z_1);\ 3(-x_1, y_1, -z_1);\ 4(-x_1, y_1, z_1)$$

$$5(x_2, y_2, z_2);\ 6(x_2, y_2, -z_2);\ 7(-x_2, y_2, -z_2);\ 8(-x_2, y_2, z_2)$$

Example1

- ***The structure of DP control system***



1. Task of determining of F_{con}o, M_{con} in earth-fixed system coordinates

2. Re-count of F_{con}o, M_{con} to body-fixed system coordinates

3.Task of Distribution of F_{con}, M_{con}

4.Re-count of body-fixed coordinates of SMO to earth-fixed system

Example2

- ***Decision of vector equations of distribution Lcon on thrusters.***
- ***Scalar form of equations of distribution for our example look as:***

$$f_{x1}+f_{x2}+f_{x3}+f_{x4}=F_{conx}$$
$$f_{y5}+f_{y6}+f_{y7}+f_{y8}=F_{cony}$$
$$f_{z1}+f_{z2}+f_{z3}+f_{z4}=F_{conz}$$
$$y_1(-f_{z1}-f_{z2}-f_{z3}-f_{z4})+z_2(f_{y5}-f_{y6}-f_{y7}+f_{y8})=M_{conx}$$
$$z_1(-f_{x1}+f_{x2}+f_{x3}-f_{x4})+x_1(f_{z1}+f_{z2}-f_{z3}-f_{z4})=M_{cony}$$
$$y_1(f_{x1}+f_{x2}+f_{x3}+f_{x4})+x_2(-f_{y5}-f_{y6}+f_{y7}+f_{y8})=M_{conz}$$

Simplify the equations by using the next designates

$$F_1=f_{x2}+f_{x3},\quad F_2=f_{x1}+f_{x4},\quad F_3=f_{z1}+f_{z2},\quad F_4=f_{z3}+f_{z4},$$
$$F_5=f_{y5},\quad F_6=f_{y6},\quad F_7=f_{y7},\quad F_8=f_{y8}$$

Then

$$F_1+F_2=F_{conx}\qquad F_3+F_4=F_{conz}\qquad z_1(F_1-F_2)+x_1(F_3-F_4)=M_{cony}$$
$$F_5+F_6+F_7+F_8=F_{cony}\qquad F_5-F_6-F_7+F_8=\frac{1}{z_2}(M_{conx}+y_1F_{conz})\qquad -F_5-F_6+F_7+F_8=\frac{1}{x_2}(M_{conz}-z_1F_{conx}).$$

Example3

•***Decision of the task of distribution .*** There are 8 unknown forces and 6 equations.

We can express F1,F2,F3 as functions of F4, analogous F5,F6,F7 as functions of F8

$$F_1=\frac{1}{2z_1}(M_{cony}+2x_1F_4-x_1F_{conz}+z_1F_{conx}),\quad F_2=F_{conx}-\frac{1}{2z_1}(M_{cony}+2x_1F_4-x_1F_{conz}+z_1F_{conx}),\quad F_3=F_y-F_4,$$

$$F_5=\frac{1}{2}[F_{cony}+\frac{1}{z_2}(M_{conx}+y_1F_{conz})]-F_8,\quad F_6=-\frac{1}{2}[\frac{1}{x_2}(M_{conz}-y_1F_{conx})+\frac{1}{z_2}(M_{conx}+y_1F_{conz})]+F_8,$$

$$F_7=\frac{1}{2}[F_{cony}+\frac{1}{x_2}(M_{conz}-y_1F_{conx})]-F_8$$

From condition $\min\limits_{F_4,F_8} J=\min\limits_{F_4,F_8}\frac{1}{2}\sum_{i=1}^{8}\lambda_i F_i^2$ follows

$$F_4=\frac{\lambda_3F_{conz}+\lambda_2\frac{x_1}{z_1}[F_{conx}-\frac{1}{2z_1}(M_{cony}-x_1F_{conz}+y_1F_{conx})]-\lambda_1\frac{x_1}{2z_1^2}(M_{cony}-x_1F_{conz}+z_1F_{conx})}{(\lambda_1+\lambda_2)\frac{x_1^2}{z_1^2}+\lambda_3+\lambda_4}$$

$$F_8=\frac{\lambda_5[F_{cony}+\frac{1}{y_2}(M_{conx}+y_1F_{conz})]+\frac{\lambda_6}{2}[\frac{1}{x_2}(M_{conz}-y_1F_{conx})+\frac{1}{z_2}(M_{conx}+y_1F_{conz})]+\frac{\lambda_7}{2}[F_{cony}+\frac{1}{x_2}(M_{conz}-y_1F_{conx})]}{\lambda_5+\lambda_6+\lambda_7+\lambda_8}$$

Example4

- ***Results of simulation for SMO***

  with Displacement 800 tonns, thruster traction limited by 4E4 N.

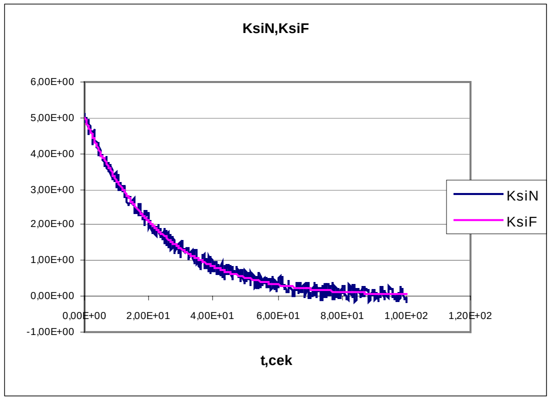
  Managing forces and moments are defined by

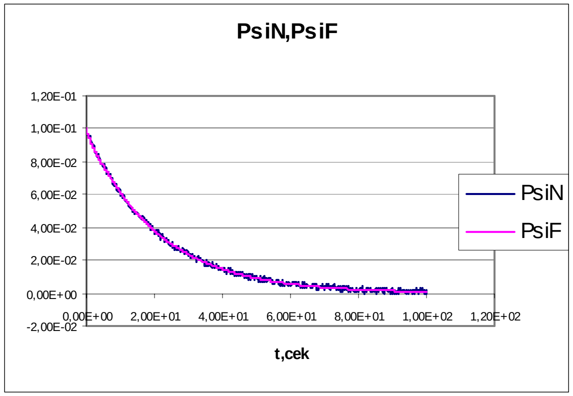
$$K_1 = diag\ (k_{1i})M\ , K_2 = diag\ (k_{2i})M\ , \quad k_{1i} = 20, k_{2i} = 1, \quad t_{tp} = 60c.$$

It is supposed presence of uniformly noise on interval ± 1 m for linear and $\pm 0.1^{\circ}$ for angular coordinates. Allowable DP errors is provided at

$$\overline{K}_2 = diag\{1,1,1,1E4,10,1E4\} \quad \overline{K}_1 = diag\{0.05;0.05;0.05;500;0.5;500\}$$

Results of filtration of signals

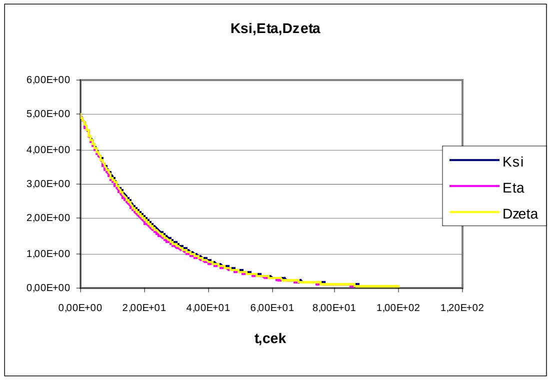


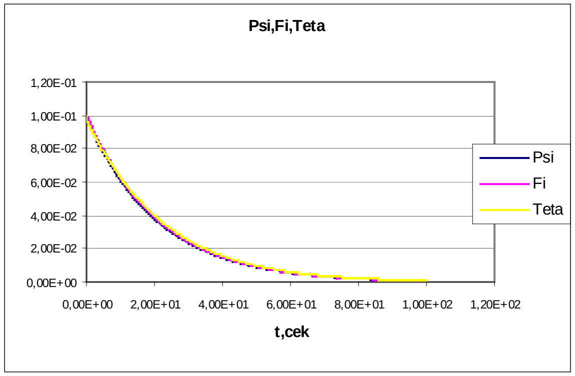


Example5

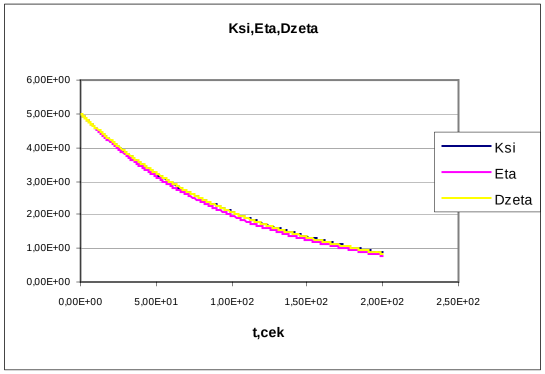
- ***Results of simulation of control motion***
- ***On this figures are shown results of simulation control processes with:***

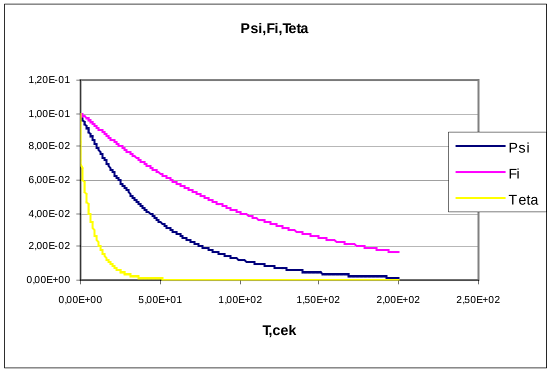
Initial deviations on linear coordinates - 5 m, on angular - 0.1 rad. and nonlimited tractions of thrusters





Tractions of thrusters are limited by 4E4 N.

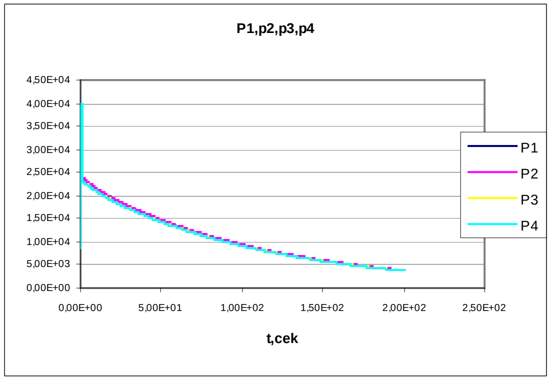


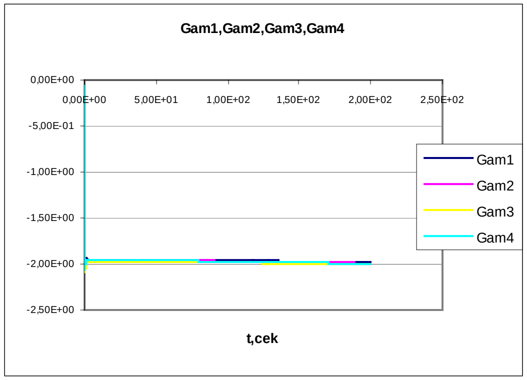


Example6

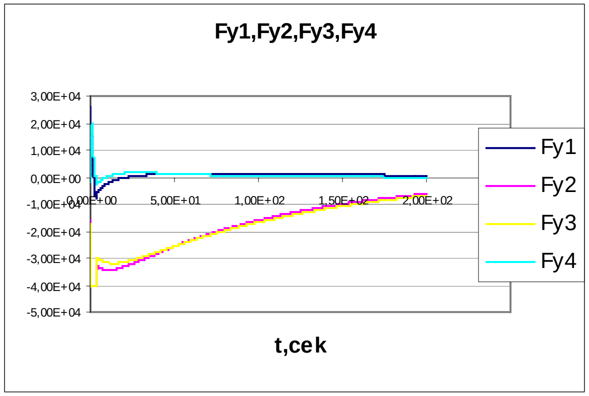
- ***Results of simulation of tractions***

The first (horizontal) group : tractions and angles of turn





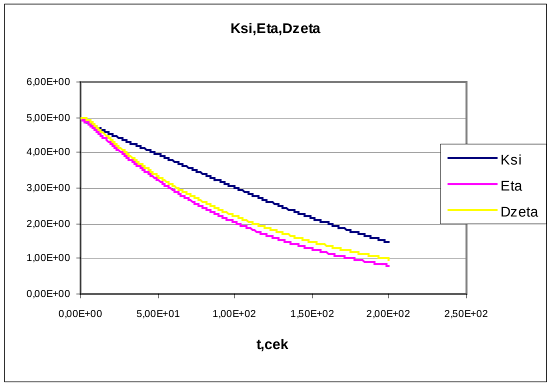
The second group : tractions

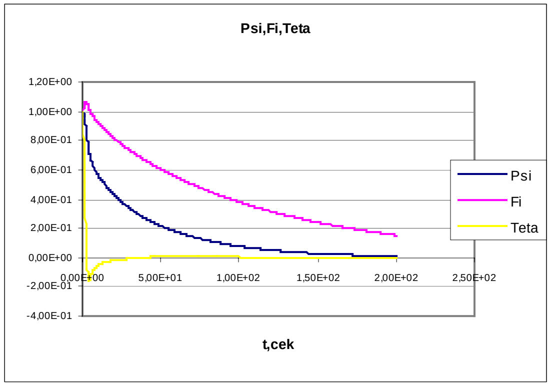


Example7

•***Results of simulation of control motion***

Initial deviations on linear coordinates - 5 m, on angular - 1 rad.





Control system of DP is stable for significant deviations of angular coordinates

Conclusions

- ***Main results***

1. The method of construction of regulators ensuring exponential stability of dynamic positioning SMO is offered.
2. The method includes of construction of control laws and observer, which provides exponential stable spatial positioning SMO in the given area of initial deviations (top level of control) and the algorithm of distribution of the received control forces and moments on the thrusters.
3. The simulation has shown high efficiency of the offered approach for DP - tasks

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